Algorithmic Aversion and Robo-Advisors

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Cherry Blossom Financial Education Institute, 2023

Research Agenda on Robo-Advising

1. **Common Perception:**
   Robo-advising = automated advice for portfolio allocation

wealthfront

Betterment

FutureAdvisor

Schwab Intelligent Portfolios

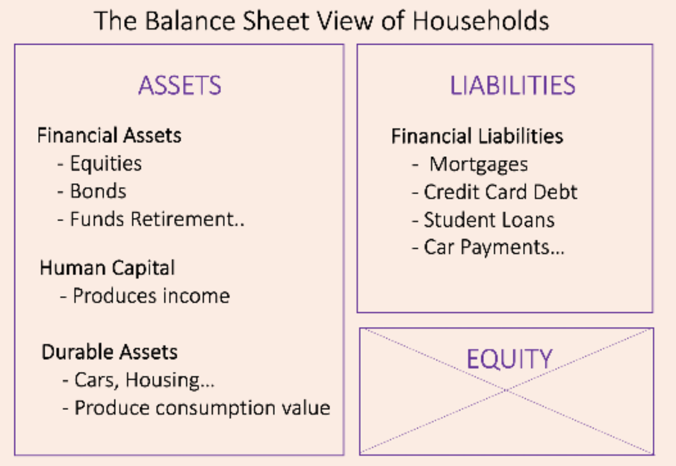
Vanguard

SIGFIG

PERSONAL CAPITAL

Research Agenda on Robo-Advising

- BUT households' decisions are more complex!
  Robo-Advising: automated advice for ANY household choice



(D'Acunto and Rossi, 2021)

Research Agenda on Robo-Advising

Robo-advising for Investment Decisions

- "*Robo-advising*," D'Acunto & Rossi
- "*The Promises and Pitfalls of Robo-advising*," D'Acunto, Prabhala & Rossi
- "*Who Benefits from Robo-advising? Evidence from Machine Learning*" Rossi & Utkus
- "*The Needs and Wants in Financial Advice: Human vs Robo-Advising*," Rossi&Utkus
- "*Algorithmic Aversion: Theory and Evidence from Robo-advice*," Ramadorai et. al

Robo-advising/FinTech for Consumption, Saving, Debt & Lending

- "*New Frontiers of Robo-Advising: Consumption, Saving, Debt, and Taxes,*" D'Acunto and Rossi
- "*Crowdsourcing Peer Information to Change Spending,*" D'Acunto, Rossi & Weber
- "*Goal Setting and Saving in the FinTech Era*" Gargano & Rossi
- "*How Costly Are Cultural Biases? Evidence from FinTech*" D'Acunto, Ghosh & Rossi
- "*Improving Households' Debt Management with Robo-advising*" D'Acunto, et. al

Algorithm Aversion: Theory and Evidence from Robo-advice

Fiona Greig Tarun Ramadorai Alberto G Rossi
Steve Utkus Ansgar Walther

Motivation

- Algorithms are ubiquitous, potentially bringing huge benefits across household markets.
- **BUT** lack of trust in algorithms may limit the scalability of tech-enabled innovation
- Algorithm aversion:
  - Barrier to tech adoption, repeatedly confirmed in psych lab findings (Dietvorst et al., 2015).
  - Little consensus about psychological underpinnings (Burton et al., 2020).
- Are there *natural limits* to the algo-adoption arising from human preferences and beliefs?

This paper:

- Understanding the different components of algorithm aversion.
- Quantifying their relative importance using the important setting of hybrid robo-advice.

This Paper

- What drives algorithm aversion?
  - Theory: Preference- and belief-based aversion; Structural estimation: Investment robo-advice

1. Model: Dynamic Model of Automation Adoption
   - Robo-advisor with uncertain performance
   - Algo aversion: *disutility*, *uncertainty*, *pessimism*
2. Data: US Hybrid robo-advisor
   - Robo-portfolio plus quasi-random assignment to high/low-“type" human advisors.
3. Reduced-form: Causal impact of the human component
   - High-type human advice reduces base quit rate by 23%
   - Value of human advice especially evident in bad market conditions
4. Structural inference
   - Ongoing disutility (preference) more important than pessimism (belief).
   - Human advice reduces uncertainty by $> 90\%$

General Model of Automation Adoption

NOTE:
We also provide a version of this general model for a Campbell-Viceira style portfolio choice problem featuring an investor with log utility of terminal wealth, who can sign-up for robo-advice, and faces disutility when interacting with an algorithm.

Link to Portfolio Choice Model

General Model of Automation Adoption

- Investors indexed by $i = 1, \ldots, I$ can use automated service for up to T periods.
- Investors quit at (endogenously) chosen time δ_i, where $\delta_i \in \{1, \ldots, T\}$.
- Quality of the service: $\theta \in \mathbb{R}$.

- Investors randomly assigned to advisors/experts indexed by $j = 1, \ldots, J$, who affect:
  1. clients' utility from consuming the service
  2. clients' beliefs at sign-up ($t = 0$): $\theta \sim N\left(m_0^j, \frac{1}{\tau_0^j}\right)$

General Model of Automation Adoption

- Post-enrollment, $t \in \{1, \ldots, \delta_i\}$, learning from performance:

$$y_t^i = \theta + u_t^i \qquad \textit{with} \qquad u_t^i \sim N\left(0, \frac{1}{\tau_y}\right)$$

- At date t, beliefs about θ updated using Bayes' rule using $\{y_1^i, \ldots, y_t^i\}$.
- Beliefs about θ are updated using the Kalman filter:

$$\theta \sim N\left(m_t^{i,j}, \frac{1}{\tau_t^j}\right)$$

$$m_t^{i,j} = m_{t-1}^{i,j} + \tfrac{\tau_y}{\tau_{t-1}^j + \tau_y}\left(y_t^i - m_{t-1}^{i,j}\right) \qquad \& \qquad \tau_t^j = \tau_0^j + t\tau_y$$

Note: depends on both i and j

General Model of Automation Adoption

Bellman Equation for the investor's problem:

$$V_t^j(m) = \max\left\{u^j(m) + \hat{\mathbb{E}}^j\left[V_{t+1}\left(m'\right)\middle| m\right], 0\right\}$$

- $V_t^j(m)$: continuation value of client still enrolled at date t with expectations $m_t^{i,j} = m$
- $u^j(m)$: expected flow of utility for a client who matches with expert j
- $\hat{\mathbb{E}}^j[V_{t+1}(m')| m]$: client's expected continuation value if enrolled until $t+1$
- Utility investor can obtain outside the service normalized to “0"

General Model of Automation Adoption

- For $t < T$, optimal to quit on date t, i.e. $\delta_i = t$, if and only if:

$$u^j(m) + \hat{\mathbb{E}}^j \left[V_{t+1}(m') \middle| m \right] \leq 0.$$

- For structural estimation, we add implementation error: $\xi_t^i \sim N\left(0, \frac{1}{\tau_\xi}\right)$ (generates cross-sectional variation in quit rates conditional on performance)
- $\Rightarrow$ For $t < T$, $\delta_i = t$ if and only if:

$$u^j\left(m - \xi_t^i\right) + \hat{\mathbb{E}}^j \left[V_{t+1}(m') \middle| m \right] \leq 0.$$

Structural Mapping

Structural Estimation

- Ultimate Goal: Estimate all parameters from simulated dynamics
- This draft: Analytical estimation equation for three-period model: $T = 2$
- Advantage: Clarifies parameter identification
- Sign up at $t = 0$. $\delta_i = t$ if and only if:

$$u^j\left(m_1^{i,j} - \xi_1^i\right) + \hat{\mathbb{E}}^j \underbrace{\left[V_2\left(m_2^{i,j}\right)\middle| m_1^{i,j}\right]}_{\equiv 0} \leq 0$$

$$\Rightarrow u^j\left(m_1^{i,j} - \xi_1^i\right) \leq 0$$

$$\Rightarrow u^j\left(m_0^{i,j}\frac{\tau_0^j}{\tau_0^j + \tau_y} + \frac{\tau_y}{\tau_0^j + \tau_y} y_1^i - \xi_i^i\right) \leq 0$$

Structural Estimation

Define $\phi^j \equiv \left(u^j\right)^{-1}(0)$:

- Critical value for quality that makes investor i indifferent between quitting and continuing.
- Interpretable as the fixed cost/disutility of participation when matched with expert j.

$\Rightarrow$ Equivalent quitting condition:

$$\underbrace{\phi^j - \frac{\tau_0^j}{\tau_0^j + \tau_y} m_0^j}_{\text{Baseline quit rate}} - \underbrace{\frac{\tau_y}{\tau_0^j + \tau_y}}_{\text{Sensitivity}} y_1^i + \xi_1^i \geq 0$$

- *Baseline quit rate* $\uparrow$ if
  - Disutility of participation increases: $\phi^j \uparrow$
  - Prior belief about robo-performance decreases: $m_0^j \downarrow$
- *Sensitivity* $\uparrow$ if
  - precision of the performance signal increases: $\tau_y \uparrow$
  - precision of the prior decreases: $\tau_0^j \downarrow$

Structural Estimation

Empirical estimation of the model

1. Group advisors into high retention (H) and low retention (L) using historical performance
2. Estimate separately preferences and belief parameters for investors assigned to
   - high-retention advisors: $\left\{\phi^H, m_0^H, \tau_0^H\right\}$
   - low-retention advisors: $\left\{\phi^L, m_0^L, \tau_0^L\right\}$

For causal interpretation:

1. Assignment of client i to human expert j should be independent of $\left\{\xi_t^i, u_t^i\right\}$.
2. Make sure the advisor-type measure is not mechanically related to clients' attrition

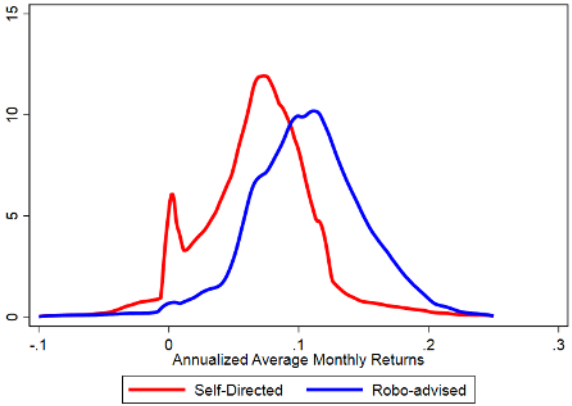
   $\rightarrow$ use leave-one-out estimator throughout our analysis

Data

Advised Investor Characteristics

	N	mean	sd	p25	p50	p75
Panel A. Demographic Characteristics						
Age	54,325	63.8	12.1	57	65	72
Male	54,744	0.6	0.5	0.0	1.0	1.0
Tenure	54,744	13.5	9.1	3.8	13.7	20.2
Panel B. Portfolio-Related Characteristics						
Wealth	54,744	$758,378	$821,029	$210,800	$478,929	$981,330
NumAssets	54,744	7.95	4.91	5.0	6.0	9.0
PctVGProducts	54,744	0.97	0.07	1.0	1.0	1.0
Panel C. Asset Allocation Characteristics						
PctMutualFunds	54,744	0.952	0.102	0.960	1.000	1.000
PctCash	54,744	0.018	0.046	0.000	0.000	0.008
PctETF	54,744	0.008	0.030	0.000	0.000	0.000
PctStocks	54,744	0.014	0.045	0.000	0.000	0.000
PctBonds	54,744	0.000	0.002	0.000	0.000	0.000
Panel D. Characteristics of Mutual Funds Held						
AcctIndex	54,744	0.828	0.178	0.745	0.858	1.000
MgtFee	54,717	0.072	0.024	0.059	0.064	0.075
ExpRatio	54,707	0.093	0.027	0.078	0.083	0.096
TurnRatio	54,685	0.268	0.120	0.190	0.280	0.337

Performance of Robo-advised and Self-directed Investors



- Robo assigns 5 glide paths based on objectives / horizon /demographics
  - Mostly 4 indexed mutual funds: VTSAX, VTIAX, VBTLX, VTABX
  - 70% cross-section of returns explained by investors' age
- 5 Principal Components: 80% of the variation in equity share
- Clustering: 96% of investors assigned to 2 glide-paths

Measuring Advisor Type

Investors with Assets above $500K are assigned to an advisor

Revealed preference approach to measuring advisor type using a leave-one-out estimator

Retention rate of advisor j and client i is:

- Ratio of clients assigned to advisor j that do not quit *excluding* from the computations:
  - Client i
  - The date t at which client i quits (takes care of cross-sectional correlation in attrition)
- Results are robust to:
  - Splitting dataset in half
  - Different specifications for controlling cross-sectional correlation in attrition

Clients' Assignment to Advisors-I

- During onboarding, clients are asked for the characteristics
- The roboadvisor generates a financial plan

- To complete sign-up, investors have to meet with an advisor
- Investors give their availability
- A scheduling system tracks advisors' availability

- All advisors have the same target number of clients
- Advisor Managers determine advisors' onboarding rate

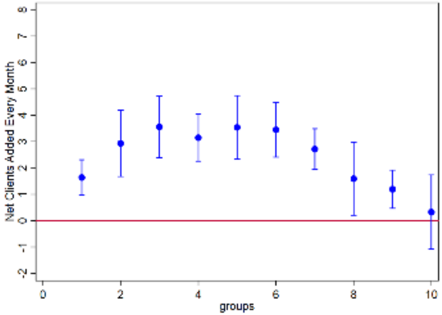
⇒ **Assumption:**
Investors' assignment to Advisors is quasi-random, conditional on factors driving onboarding.

Validating Quasi-Random Assignment-I

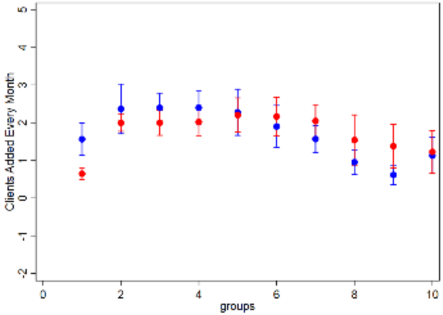
	High Retention		Low Retention		Diff		
	mean	N	mean	N	mean	t-stat	N
Age	64.540	24,514	65.657	23,511	1.117***	(3.33)	48,025
Male	0.577	25,739	0.618	24,085	0.041***	(4.76)	49,824
Tenure	14.666	25,739	15.635	24,085	0.969***	(3.03)	49,824
Wealth	946,754	25,739	993,861	24,085	47,107	(0.39)	49,824
NumAssets	10.717	25,739	11.438	24,085	0.721	(1.69)	49,824
PctVGProducts	0.853	25,706	0.850	24,062	-0.003	(-0.81)	49,768
PctMutualFunds	0.666	25,706	0.672	24,062	0.006	(0.61)	49,768
PctCash	0.234	25,706	0.226	24,062	-0.007	(-0.67)	49,768
PctETF	0.035	25,706	0.034	24,062	0.000	(-0.27)	49,768
PctStocks	0.046	25,706	0.047	24,062	0.001	(0.84)	49,768
PctBonds	0.002	25,706	0.002	24,062	0.000**	(2.65)	49,768
AcctIndex	0.436	25,739	0.438	24,084	0.002	(0.14)	49,823
MgtFee	0.147	23,877	0.147	22,931	0.001	(0.28)	46,808
ExpRatio	0.209	23,299	0.206	22,396	-0.003	(-0.18)	45,695
TurnRatio	0.328	22,918	0.343	21,787	0.016**	(2.16)	44,705
Ret. Pre-PAS	0.051	22,040	0.045	20,884	-0.005	(-0.98)	42,924
Adj. Ret. Pre-PAS	-0.007	22,040	-0.009	20,884	-0.002	(-1.64)	42,924

⇒ Investors assigned to low- and high-retention advisors are virtually indistinguishable

Validating Quasi-Random Assignment-II



(a) All Advisors

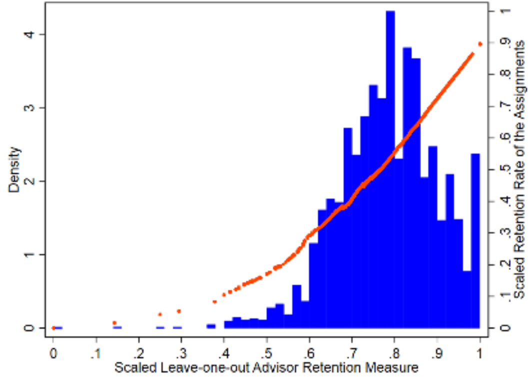


(b) High Retention; Low Retention

⇒ High- and Low-retention advisors are assigned clients at the same rate

Empirical Estimates

Heterogeneity in Client Retention and Advisor Type



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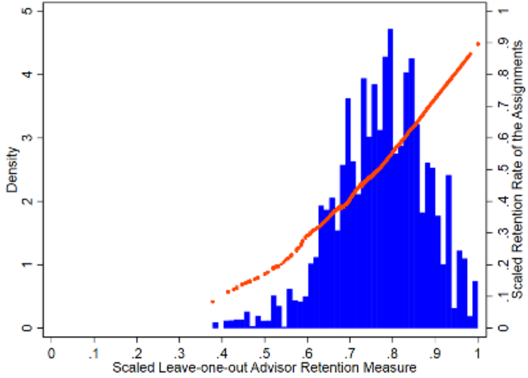


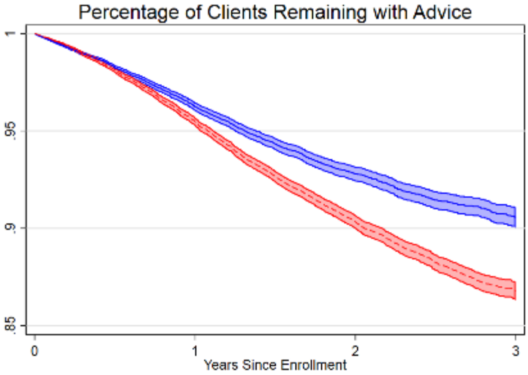
(b) Excluding 10% of Advisors with Few clients

- Advisors have different (scaled) client retention in the cross-section
- But advisor fixed effect has no explanatory power for returns

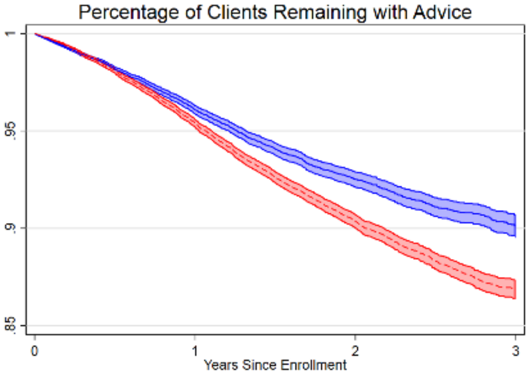
Non Parametric Survival Estimates-I

- Compute advisors' retention using leave-one-out estimator
- Split them into two groups:
  - Type 0: advisor with leave-one-out retention below median
  - Type 1: advisors with leave-one-out retention above median
- Take all investors signing up for PAS
- Estimate $\widehat{S}(t) = \Pi_{i:t_i \leq t}\left(1 - \frac{d_i}{n_i}\right)$ for each group
  where:
  - t_i: time when at least one investor quits
  - d_i: number of clients quitting robo-advice at time t
  - n_i: number of clients who have stayed with robo-advice

Non Parametric Survival Estimates-II



(a) All Advisors



(b) Excluding 10% of Advisors with Few clients

- Cox Model: clients assigned to Type-1 human advisors have 25.4% lower hazard

Effect of Human Advice Across Market Conditions-I

Regression Results:

$$Dummy\_quit_{i,t} = \alpha + \beta\, I_{\{MKT\_RET_{t-1}<0\}} + \gamma\, I_{\{Type1\_Advisor_i=1\}} + \delta\, I_{\{MKT\_RET_{t-1}<0\}} \times I_{\{Type1\_Advisor_i=1\}} + \epsilon_{i,t},$$

where

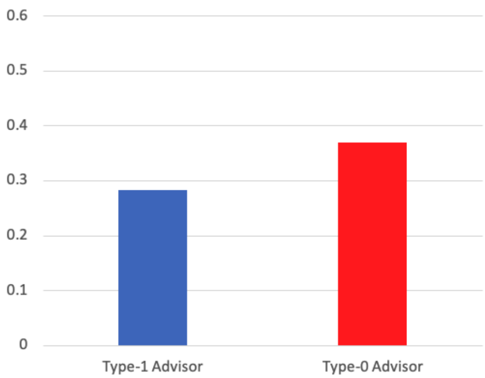
- $Dummy\_quit_{i,t}$: 1 if investor i quits in month t;
- $I_{\{MKT\_RET_{t-1}<0\}}$: 1 if market returns are negative in month t-1
- $I_{\{Type1\_Advisor_i=1\}}$: 1 if investor i is assigned to a high-retention advisor

Effect of Human Advice Across Market Conditions-II

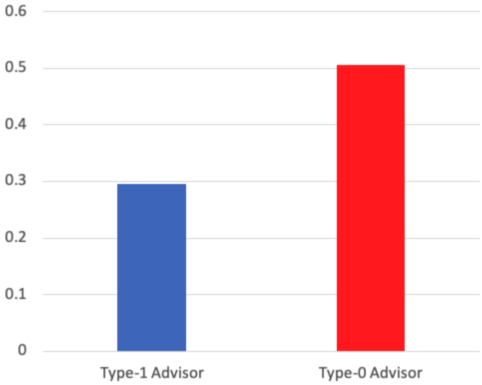
	CRSP Return	Investor Return	CRSP Volatility	Investor Volatility
Bad Market	0.136*** (2.98)	0.145*** (3.96)	0.125*** (3.20)	0.117*** (4.02)
Type1_Advisor	-0.086*** (-4.74)	-0.078*** (-4.03)	-0.051*** (-3.14)	-0.056*** (-4.41)
Interaction	-0.123*** (-3.77)	-0.123*** (-3.90)	-0.102*** (-3.21)	-0.091*** (-3.61)
Constant	0.369*** (17.70)	0.359*** (16.23)	0.325*** (17.78)	0.328*** (17.87)
Clustering	Date&User	Date&User	Date&User	Date&User
R-square	0.00011	0.00013	0.00014	0.00013
N	938,314	938,314	938,314	938,314

Effect of Human Advice Across Market Conditions-III

Using Column (1) coefficients:



(a) Good Market Conditions



(b) Poor Market Conditions

→ Low Retention advisors lose more clients in bad markets

→ High Retention advisors perform similarly across market conditions

Structural Mapping—Empirics-I

- Baseline quit rates:

$$\widehat{\phi^L - \frac{\tau_0^L}{\tau_0^L+\tau_y} m_0^L} = 0.369 * 12 = 4.43\%$$

$$\widehat{\phi^H - \frac{\tau_0^H}{\tau_0^H+\tau_y} m_0^H} = (0.369 - 0.086) * 12 = 3.40\%$$

$\Rightarrow$ High-type advisor reduces baseline quit rates by $1 - 3.4/4.43 = 23.25\%$

- Sensitivity to performance:

$$\widehat{\frac{\tau_y}{\tau_0^L+\tau_y}} = 0.136 * 12 = 1.63\%$$

$$\widehat{\frac{\tau_y}{\tau_0^H+\tau_y}} = (0.136 - 0.123) * 12 = 0.16\%$$

$\Rightarrow$ High-type advisor reduces sensitivity to performance by $1 - 0.16/1.63 = 90.18\%$

Structural Mapping—Empirics-II

- Does sensitivity vary with tenure?

	Short Tenure	Long Tenure
Bad Market	0.219***	0.044
	(7.69)	(1.36)
Type1_Advisor	-0.057***	-0.089***
	(-3.29)	(-3.62)
Interaction	-0.158***	-0.102**
	(-3.55)	(-2.19)
Constant	0.302***	0.417***
	(23.77)	(16.95)

$$\textit{Long tenure investors}: \overbrace{\phi^L - \underbrace{\frac{\tau_0^L}{\tau_0^L + \tau_y} m_0^L}_{\simeq 0 \text{ (convergence)}}} \simeq \widehat{\phi^L} = 0.417 * 12 = 5.00\%$$

$$\overbrace{\phi^H - \underbrace{\frac{\tau_0^H}{\tau_0^H + \tau_y} m_0^H}_{\simeq 0 \text{ (convergence)}}} \simeq \widehat{\phi^H} = (0.417 - 0.089) * 12 = 3.94\%$$

$\Rightarrow$ High-type advisor reduces baseline fixed cost/disutility by $1 - 3.94/5 = 21.2\%$

Conclusions

- We present a model of algorithm aversion featuring:
  - Learning about algorithm's ability
  - Ongoing disutility from using the algorithmic solution
- We map the model to the real world using
  - Data from a hybrid robo-advisor (PAS)
  - Quasi-random assignment of clients to advisors
- Main findings
  - Significant algo aversion reduced by human advisors
  - High-type advisors retain more clients in turbulent times
  - Experienced clients react less to market turbulence
  - Uncertainty and disutility channels of algorithm aversion are structurally most important

Appendix Slides

Model Specialized to Portfolio Choice and Robo-advice-I

- Investor can allocate fraction $\alpha_t \geq 0$ to a robo-advisor. $1 - \alpha_t$ invested outside portfolio.
- Client's utility is $w_T = \ln(W_T)$, where W_T is the final wealth.
- When robo-advised, client suffers a fixed cost f^j: depends on the identity of human advisor.
- Log return on the outside portfolio is deterministic and given by $\bar{r}$.
- Robo-advisor generates stochastic log returns given by $r^i_{t+1} = \bar{r} + \theta + u^i_{t+1}$.
- Investor's beliefs about θ, as a function of her human advisor j, are as in the general model.
- Investor's log wealth, evolves according to the following approximate law of motion:

$$w^i_{t+1} - w_t \simeq \bar{r} + \alpha_t y^i_{t+1} + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t), \quad where \quad \sigma^2 = 1/\tau_y \qquad (1)$$

Proof

Back

Model Specialized to Portfolio Choice and Robo-advice-II

- Conjecture that the investor's continuation value, if still enrolled at date $t < T$, is:

$$F_t(w, m) = w + (T - t)\,\bar{r} + V_t(m)\,.$$

- At date t, two options:
  - Quit $\Rightarrow$ final utility takes the deterministic value $w + (T - t)\,\bar{r}$.
  - Stay robo-advised with an optimally chosen portfolio weight α_t.
- Investor's Bellman Equation:

$$F_t(w, m) = \max \left\{ w + (T - t)\,\bar{r}, -f^j + \max_{\alpha \geq 0} \hat{\mathbb{E}}^j \left[F_{t+1}\left(w', m'\right) | w, m, \alpha_t = \alpha \right] \right\} \quad (2)$$

Back

Model Specialized to Portfolio Choice and Robo-advice-III

- Substitute conjecture and law of motion of wealth to write the last term of Equation (**??**) as:

$$\begin{aligned} \hat{\mathbb{E}}^j\left[F_{t+1}\left(w', m'\right)|w, m, \alpha_t = \alpha\right] &= w' + (T - (t+1))\,\bar{r} + \hat{\mathbb{E}}^j\left[V_t\left(m'\right)|m\right] \\ &= w + (T - t)\,\bar{r} + \alpha m + \frac{1}{2}\sigma^2\alpha\,(1-\alpha) + \hat{\mathbb{E}}^j\left[V_t\left(m'\right)|m\right] \qquad (3) \end{aligned}$$

  where we use that: $m = \hat{\mathbb{E}}^j\left[y'|m\right]$ by definition

- Inner maximization in Equation (**??**) is solved by optimal portfolio weight

$$\hat{\alpha} = \max\left\{\frac{m + \frac{1}{2}\sigma^2}{\sigma^2}, 0\right\},$$

Back

Model Specialized to Portfolio Choice and Robo-advice-IV

The value of (**??**) is:

$$E\left[F_{t+1}\left(w',m'\right)|w,m,\alpha_t=\hat{\alpha}\right]=w+\left(T-t\right)\bar{r}+\left(m+\frac{1}{2}\sigma^2\right)\hat{\alpha}-\frac{1}{2}\sigma^2\hat{\alpha}^2+\hat{\mathbb{E}}^j\left[V_{t+1}\left(m'\right)|m\right]$$
$$=w+\left(T-t\right)\bar{r}+\hat{\mathbb{E}}^j\left[V_{t+1}\left(m'\right)|m\right]+\frac{1}{2}\left[SR\left(m\right)\right]^2,$$

where

$$SR\left(m\right)=\begin{cases}0, & m+\frac{1}{2}\sigma^2<0,\\ \frac{m+\frac{1}{2}\sigma^2}{\sigma}, & \text{otherwise.}\end{cases}$$

Substitute this into (**??**), together with the conjecture solution, to obtain:

$$V_t\left(m\right)=\max\left\{\underbrace{-f^j+\frac{1}{2}\left[SR\left(m\right)\right]^2}_{\equiv u^j(m)}+\hat{\mathbb{E}}^j\left[V_{t+1}\left(m'\right)|m\right],0\right\},$$

which maps into our general model when the investor's utility function is:

$$u^j\left(m\right)=-f^j+\frac{1}{2}\left[SR\left(m\right)\right]^2.$$

Back

Derivation of Intertemporal Budget Constraint-I

Here we provide the derivation of the law of motion of wealth, i.e., Equation (**??**).

- Consider continuous-time model where per-unit value P_t of robo portfolio follows:

$$\frac{dP_t}{P_t} = \left(\bar{r} + \theta + \frac{1}{2}\sigma^2\right) dt + \sigma dZ_t,$$

where Z_t is a standard Brownian Motion

- Discrete-time representation:

$$r_{t+1} \equiv \log\left(\frac{P_{t+1}}{P_t}\right) = \bar{r} + \theta + \sigma u_{t+1}, \; where \; u_{t+1} = Z_{t+1} - Z_t$$

- Outside portfolios evolves according to

$$\frac{dB_t}{B_t} = \bar{r} dt$$

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Derivation of Intertemporal Budget Constraint-II

- Investor's wealth evolves according to:

$$\begin{aligned}\frac{dW_t}{W_t} &= \alpha_t \frac{dP_t}{P_t} + (1-\alpha_t)\frac{dB_t}{B_t} \\ &= \alpha_t\left[\left(\bar{r}+\theta+\frac{1}{2}\sigma^2\right)dt + \sigma dZ_t\right] + (1-\alpha_t)\,\bar{r}dt \\ &= \left[\alpha_t\left(\bar{r}+\theta+\frac{1}{2}\sigma^2\right) + (1-\alpha_t)\,\bar{r}\right]W_t dt + \alpha_t \sigma W_t dZ_t\end{aligned}$$

- Converting to log returns, and applying Ito's lemma to $f(W) = logW$, we obtain:

$$\begin{aligned}d\log W_t = df(W_t) &= f'(W_t)\,dW_t + \frac{1}{2}f''(W_t)\,(dW_t)^2\,dt \\ &= \left[\alpha_t\left(\bar{r}+\theta+\frac{1}{2}\sigma^2\right) + (1-\alpha_t)\,\bar{r}\right]dt + \alpha_t\sigma dZ_t - \frac{1}{2}(\alpha_t\sigma)^2\,dt \\ &= \left[\alpha_t(\bar{r}+\theta) + (1-\alpha_t)\,\bar{r} + \frac{1}{2}\sigma^2\alpha_t(1-\alpha_t)\right]dt + \alpha_t\sigma dZ_t\end{aligned}$$

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Derivation of Intertemporal Budget Constraint-III

- For discrete time approximation, set $dt = 1$ to get:

$$\begin{aligned}
\log W^i_{t+1} - \log W_t &= \alpha_t (\bar{r} + \theta) + (1 - \alpha_t)\,\bar{r} + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t) + \alpha_t \sigma u_{t+1} \\
&= \alpha_t (\bar{r} + \theta + \sigma u_{t+1}) + (1 - \alpha_t)\,\bar{r} + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t) \\
&= \alpha_t r_{t+1} + (1 - \alpha_t)\,\bar{r} + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t) \\
&= \bar{r} + \alpha_t (r_{t+1} - \bar{r}) + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t) \\
&= \bar{r} + \alpha_t y_{t+1} + \frac{1}{2}\sigma^2 \alpha_t (1 - \alpha_t),
\end{aligned}$$

which establishes Equation (**??**), where we have again used $u_{t+1} = Z_{t+1} - Z_t$.

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