Beliefs, Stockholding, and Wealth Accumulation Throughout the Life Cycle

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Outline

1. Motivation and Question

2. Empirical Facts

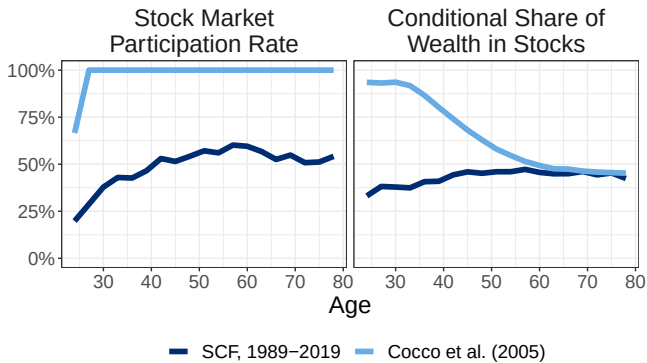
3. Model

4. Preliminary Estimation Exercise

5. Conclusions & Future Work

Life-Cycle Portfolio Models

▸ Why study LCP models



- It is challenging to model life-cycle stockholding.
- More challenging to simultaneously model wealth.

Deviations From Full-Information Rational-Expectations

Advances in the financial-literacy literature

- See Lusardi and Mitchell (2014) for a review.
- Prevalent illiteracy, heterogeneity, socioeconomic gaps, related to behavior.

Increasing quantity and quality of data on stock-market expectations

- E.g., Dominitz and Manski (2007), Arrondel, Calvo Pardo, and Tas (2014), Ameriks et al. (2020), Das, Kuhnen, and Nagel (2020), and Giglio et al. (2021).
- "Pessimism", heterogeneity, socioeconomic gaps, related to behavior.

Difficult to incorporate these insights into our models.

- Examples: Kézdi and Willis (2011) and Lusardi, Michaud, and Mitchell (2017).

This Paper

Literature Data Sources

Q: Can beliefs data help us model life-cycle stockholding and wealth?

What I do

- Estimate the distribution of people's beliefs about stocks from survey data (HRS).
  - "Pessimism", persistent heterogeneity, socioeconomic gaps.
- Use estimated beliefs in a life-cycle portfolio model.
- Estimate preferences to match wealth and stockholding (SCF).

Punchline

- Estimated beliefs improve the model's fit.
- Also, the estimated preferences move towards "usual" ranges.

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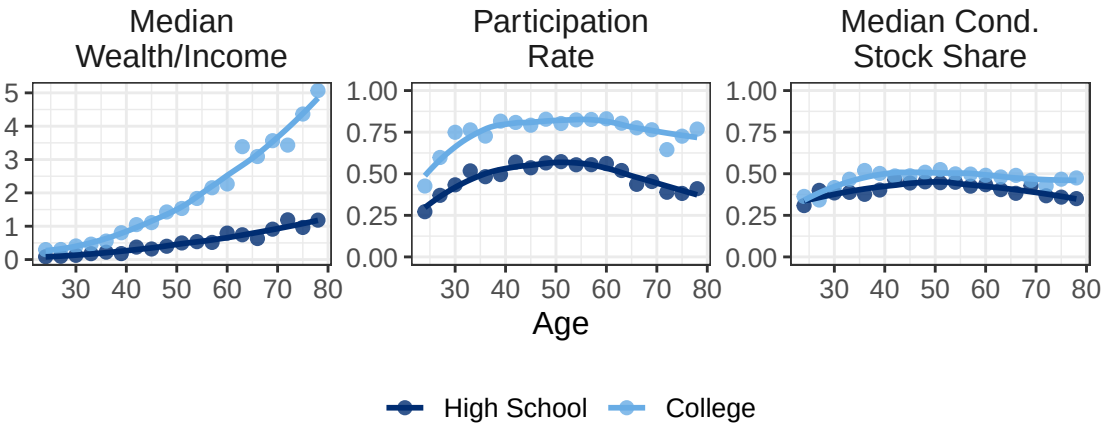
Wealth and Stockholding in the SCF, 1989-2019

Definitions



Sample: wealth ≥ 0, income > 0, $1920 \leq$ Cohort ≤ 1995.

Beliefs Data in the HRS

▸ Beliefs and Stockholding

Questions: *What is the percent chance that, by this time next year, mutual fund shares invested in a stock market index will*

- $\mathbf{P_0}$: *be worth more than they are today?*
- $\mathbf{P_{20}}$: *have gained in value by more than 20%?*

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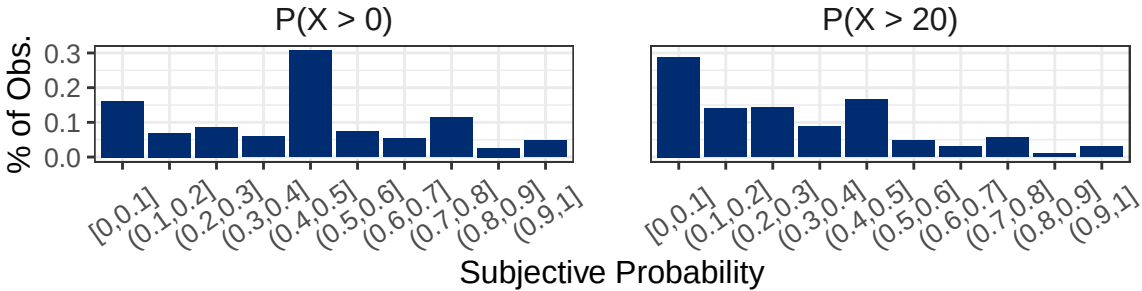

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Estimating the Distribution of Beliefs

Summ. Stats | Mistakes | Model | Coef. Estimates

Adapt model from Ameriks et al. (2020)

- Everyone believes $\ln \tilde{R} \sim \mathcal{N}(\mu_i, \sigma_i)$.
- (μ_i, σ_i) are heterogeneous and unobserved.

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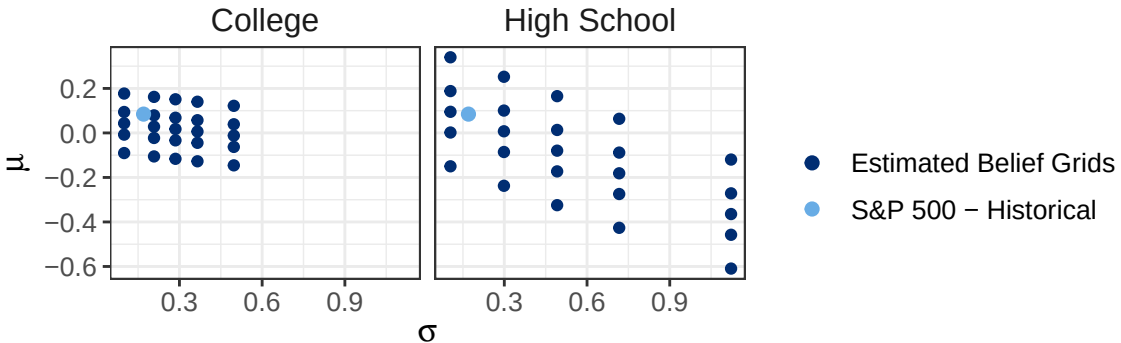

Life Cycle Model — Overview

▸ Bellman Equation ▸ Timing Diagram

Framework:

- CRRA utility of consumption.
- Finite, stochastic lives (24 to 100 years). Retirement at 65.
- Borrowing constraint.

Income:

- Education-specific calibration from Cagetti (2003). ▸ Plot
- Uninsurable shocks, permanent and transitory. ▸ Volatility

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Financial assets: ▸ Frictions

- Risk-free one-period bond.
- Risky stocks fund.

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An Initial Test

Estimation by SMM

▸ Loss Function

- Target age profiles of median wealth/income, participation rate and avg. share of wealth in stocks.
- Only pre-retirement.
- Tweak time-discount factor (β) and CRRA (ρ).

An Initial Test

Estimation by SMM

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Compare beliefs specifications

- Full-Information Rational Expectations (F.I.R.E).
- Estimated belief girds.

Preliminary Results

	High-School	College
	F.I.R.E.	
CRRA (ρ)	12.24	
	(0.08)	
Disc. Fact. (β)	0.20	
	(0.00)	
S.M.M. Loss	10.65	

Preliminary Results

	High-School		College
	F.I.R.E.	Est. Beliefs	
CRRA (ρ)	12.24	6.94	
	(0.08)	(0.08)	
Disc. Fact. (β)	0.20	0.57	
	(0.00)	(0.00)	
S.M.M. Loss	10.65	5.92	

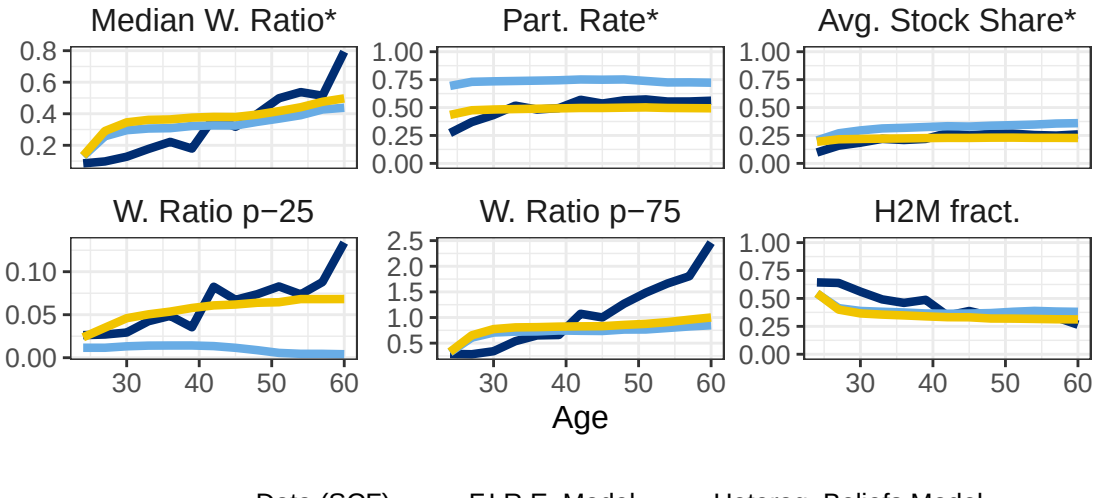
Preliminary Results

	High-School		College
	F.I.R.E.	Est. Beliefs	F.I.R.E.
CRRA (ρ)	12.24	6.94	12.86
	(0.08)	(0.08)	(0.10)
Disc. Fact. (β)	0.20	0.57	0.41
	(0.00)	(0.00)	(0.00)
S.M.M. Loss	10.65	5.92	3.48

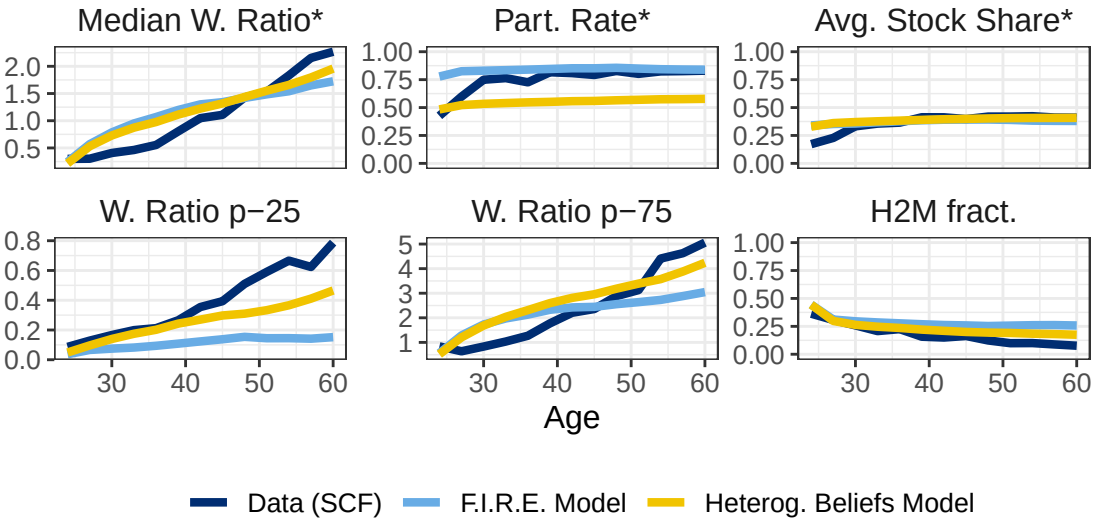
Preliminary Results

	High-School		College	
	F.I.R.E.	Est. Beliefs	F.I.R.E.	Est. Beliefs
CRRA (ρ)	12.24	6.94	12.86	2.73
	(0.08)	(0.08)	(0.10)	(0.02)
Disc. Fact. (β)	0.20	0.57	0.41	0.93
	(0.00)	(0.00)	(0.00)	(0.00)
S.M.M. Loss	10.65	5.92	3.48	3.03

High-School


Median W. Ratio*
Part. Rate*
Avg. Stock Share*
W. Ratio p−25
W. Ratio p−75
H2M fract.
Age
Data (SCF)
F.I.R.E. Model
Heterog. Beliefs Model

College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Conclusions

- Measured beliefs about the stock market are predictive of financial decisions.
  - Optimism robustly predicts extensive and intensive-margin stockholding.
  - Informative but very noisy.

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- Matching these measurements can improve our structural models:
  - Improving their fit of empirical patterns.
  - Bringing their parameter estimates to more reasonable ranges.

Conclusions

- Measured beliefs about the stock market are predictive of financial decisions.
  - Optimism robustly predicts extensive and intensive-margin stockholding.
  - Informative but very noisy.
- Matching these measurements can improve our structural models:
  - Improving their fit of empirical patterns.
  - Bringing their parameter estimates to more reasonable ranges.
- Current work:
  - More resourceful beliefs-measurement system.
  - Post retirement behavior: bequests, health shocks.

Thank you!

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Motivation

Back

Households' portfolio allocations are crucial to understanding

- Wealth distribution:
  - Heterogeneous returns (Benhabib and Bisin 2018; Fagereng, Guiso, et al. 2020).
- Cross-group differences in wealth:
  - Differences in portfolios along the wealth distribution (Kuhn, Schularick, and Steins 2020).
  - Can arise from differences in sophistication (Lusardi, Michaud, and Mitchell 2017).
- Effects of monetary and and fiscal policy:
  - Wealth level, allocation and liquidity are important for MPC out of transfers (Kaplan and Violante 2014; Carroll et al. 2017).
  - Heterogeneity is important for the response of investment (Luetticke 2021).

Improving Life-Cycle Portfolio Models

Back

Attempts to reconcile models with data:

- **Costs**: Vissing-Jorgensen (2002), F. Gomes and Michaelides (2005), Khorunzhina (2013), and Campanale, Fugazza, and F. Gomes (2015).
- **Distribution of returns**: Rietz (1988), Barro and Ursúa (2012), Fagereng, Gottlieb, and Guiso (2017), and Catherine (2021).

This paper:

Why measured beliefs?

- Transaction costs and liquidity differences between assets.
- *Heterogeneous* beliefs about returns, *estimated* from survey measurements.

Why Beliefs?

Back

- Beliefs are central to our models.
- Increasingly available data: SPF, HRS, SCE, MSC...
- Enhance assumptions with imperfect measurements.
- Evidence of their informativeness/behavioral effects:
  - **In portfolio choice:** Dominitz and Manski (2007), Arrondel, Calvo Pardo, and Tas (2014), Giglio et al. (2021), and Ameriks et al. (2020).
  - **In other contexts:** demand for insurance (Finkelstein and McGarry 2006), bequests (Gan et al. 2015), educational attainment (Papageorge, Gershenson, and Kang 2020).
- Incorporating these data to dynamic structural models brings
  - Design and technical challenges.
  - Potential to tackle issues like the stockholding puzzle.

Sources

Back

Health and Retirement Study:

- 2002 - 2016, biennial.
- Longitudinal.
- Wealth and **expectations** modules.
- Ages $\geq$ 50.

Survey of Consumer Finances:

- 1989 - 2019, triennial.
- Repeated cross-sections.
- Detailed wealth data.

Main Variable Definitions

Back Specifics

- Wealth: total financial assets (no vehicles, real state, businesses).
- Wealth ratio: Wealth/(Labor + Pension income), if income was "usual."
- Stocks: directly held, mutual funds.

Variables of interest:

- Participation: $\mathbf{1}[\text{Stocks} > 0]$.
- Stocks' share: Stocks/Wealth, if wealth > 0.
- Cond. Stock's share: stocks' share if > 0.
- Hand-to-mouth (H2M): Wealth $\leq (1/6)$ usual income (Zeldes 1989).

Variable Definitions

Back

- SCF, from their summary files.
  - Wealth (`fin`): total value of financial assets.
  - Stock holdings (`equity`): directly held, mutual funds, IRA/Keoghs (estimated).
  - Income (`wageinc + ssretinc`): wage, salary, soc. sec, and pension income.
- HRS, from the RAND files.
  - Wealth (`atotf`): net financial wealth.
  - Stock holdings (`astck`): only directly held and mutual funds.
  - Income (`HwITOT`): total income.

Beliefs and Stockholding (HRS)

- More optimistic agents do hold more stocks.

		Quantile: Stocks' Share of Wealth				
Belief $P(X > 0)$	N. Obs	50	60	70	80	90
$[0, 0.25]$	14053	0	0.00	0.00	0.18	0.58
$(0.25, 0.5]$	22451	0	0.00	0.05	0.31	0.64
$(0.5, 0.75]$	10620	0	0.05	0.23	0.45	0.72
$(0.75, 1]$	7893	0	0.06	0.25	0.48	0.73

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Evidence of measured beliefs $\sim$ stockholding:

- Dominitz and Manski (2007), Arrondel, Calvo Pardo, and Tas (2014), Drerup, Enke, and von Gaudecker (2017), Das, Kuhnen, and Nagel (2020), Ameriks et al. (2020), and Giglio et al. (2021) among others.

Beliefs and the Extensive Margin (HRS)

Back

- Known cross-sectional relationship.
- Also true within individuals.

	Dep. Var.: Participation$_t$ (LPM).		
	Model 1	Model 2	Model 3
Belief$_t$ $P(X > 0)$	0.26^{***}	0.16^{***}	0.04^{***}
	(0.01)	(0.01)	(0.01)
Belief$_t$ $P(X > 0)$: DK/Refused	-0.09^{***}	-0.05^{***}	-0.01
	(0.01)	(0.01)	(0.00)
Log-Income (HH)		0.10^{***}	0.03^{***}
		(0.00)	(0.00)
Household F.E.			✓
Educ. Dummies		✓	
Age and Year Dummies	✓	✓	✓
Gender		✓	
R^2	0.06	0.16	0.03
Adj. R^2	0.06	0.16	-0.32
Num. obs.	73754	73745	73754

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. SEs clustered at the Household level.

Beliefs and the Intensive Margin (HRS)

Back

$$\text{Share}_{i,t} = \mathbf{1}[\text{Share}^*_{i,t} \geq 0] \times \textit{Share}^*_{i,t}$$

$$\text{Share}^*_{i,t} = \alpha + \beta \text{Beliefs}_{i,t} + x'_{i,t}\gamma + u_i + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2)$$

	Dep. Var.: $\text{Share}^*_{i,t}$ (Tobit).		
	Model 1	Model 2	Model 3
Belief$_t$ $P(X > 0)$	0.43^{***}	0.26^{***}	0.14^{***}
	(0.01)	(0.01)	(0.01)
Belief: DK/Refused	-0.24^{***}	-0.16^{***}	-0.09^{***}
	(0.01)	(0.01)	(0.01)
Log-Income (HH)		0.18^{***}	0.11^{***}
		(0.00)	(0.00)
Household Rand. Effecs			✓
Educ. Dummies		✓	✓
Gender		✓	✓
Age and Year Dummies	✓	✓	✓
Num. Obs.	73745	73745	73745

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

The Effect of Experience

Back

Malmendier and Nagel (2011):

- Experienced returns are associated with participation.
- Beliefs might be a channel

	Dep. Var.: $P(\text{Participation}_{i,t} = 1)$ (Probit).				
	Model 1	Model 2	Model 3	Model 4	Model 5
Exp. ln R			12.30***		9.02*
			(3.48)		(3.52)
Belief_t				0.49***	0.49***
				(0.03)	(0.03)
Belief: DK/Refused				−0.28***	−0.28***
				(0.02)	(0.02)
Log-Income (HH)		0.39***	0.39***	0.37***	0.37***
		(0.01)	(0.01)	(0.01)	(0.01)
Age and Year Dummies	✓	✓	✓	✓	✓
Educ. and Gender	✓	✓	✓	✓	✓
Area Under ROC	0.70	0.75	0.75	0.76	0.76
Num. obs.	73745	73745	73745	73745	73745

***$p < 0.001$; **$p < 0.01$; *$p < 0.05$. SEs clustered at the Household level.

Belief Grids, Summary Statistics

Back

	Log-Return Mean μ_i		Log-Return S.D. σ_i	
Education	Mean	St. Dev.	Mean	St. Dev.
College	0.02	0.09	0.29	0.14
High School	-0.10	0.23	0.55	0.36

Challenges of Beliefs Data — Mistakes

Back

- **Mistake 1:** answering P_0 or P_{20} is 0 or 1.
- **Mistake 2:** answering $P_{20} > P_0$.

		Fraction of Respondents		
Education	**Obs.**	**Mistake 1**	**Mistake 2**	**No Mistakes**
Less than H.S.	5949	0.18	0.54	0.35
High School	24976	0.12	0.43	0.48
College	11891	0.09	0.26	0.65

- **Rounding**

	Fraction of Respondents	
Question	Multiple of 5%	Multiple of 10%
P_0	0.982	0.812
P_{20}	0.977	0.822

Probabilistic Responses à la Ameriks et al. (2020)

Back

- Everyone believes $\ln \tilde{R} \sim \mathcal{N}(\mu_i, \sigma_i)$.
- (μ_i, σ_i) are heterogeneous, unobserved, and follow

$$\begin{bmatrix} \mu_i \\ \sigma_i \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \nu_\mu \\ \nu_\sigma \end{bmatrix}, \begin{bmatrix} \omega_\mu^2 & \rho\omega_\mu\omega_\sigma \\ \rho\omega_\mu\omega_\sigma & \omega_\sigma^2 \end{bmatrix} \right) \mid \sigma_i > 0.$$

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$$\begin{bmatrix} \mu_i \\ \sigma_i \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \nu_\mu \\ \nu_\sigma \end{bmatrix}, \begin{bmatrix} \omega_\mu^2 & \rho\omega_\mu\omega_\sigma \\ \rho\omega_\mu\omega_\sigma & \omega_\sigma^2 \end{bmatrix} \right) \mid \sigma_i > 0.$$

- People answer the questions about future returns using

$$P_{i,t}^0 = \left[\Phi\left(\frac{\mu_i}{\sigma_i} + \varepsilon_{i,t}^0 \right) \right], \qquad P_{i,t}^{20} = \left[\Phi\left(\frac{\mu_i - \ln 1.20}{\sigma_i} + \varepsilon_{i,t}^{20} \right) \right]$$

- $\{\varepsilon_{i,t}^0, \varepsilon_{i,t}^{20}\}' \sim \mathcal{N}(\vec{0}, \Sigma)$ are behavioral/conditioning errors.
- $[\cdot]$ rounds to the nearest 0.1 multiple.

Beliefs Model Estimates

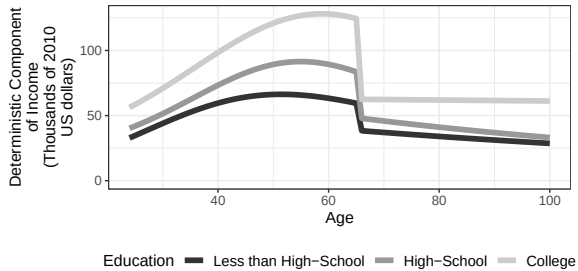
Back

	Less than H.S.	High School	College
ν_μ	11.161	0.004	0.018
ν_σ	−20.785	0.306	0.279
ω_μ	2.184	0.292	0.098
ω_σ	4.128	0.519	0.154
ρ	−0.995	−0.801	−0.218
$\sqrt{V(\varepsilon^0)}$	0.926	0.749	0.662
$\sqrt{V(\varepsilon^{20})}$	0.917	0.764	0.697
Corr.$(\varepsilon^0, \varepsilon^{20})$	0.617	0.572	0.465
Log-likelihood	−26213.947	−107739.242	−50584.745

Income Process, Deterministic Trajectories

Back

$$\ln Y_{t+1} = f(t) + \ln P_{t+1} + \ln \theta_{t+1}$$
$$\ln P_{t+1} = \ln P_t + \ln \psi_{t+1}$$
$$\ln \theta_{t+1} \sim \mathcal{N}(-\sigma_\theta^2/2, \sigma_\theta^2), \quad \ln \psi_{t+1} \sim \mathcal{N}(-\sigma_\psi^2/2, \sigma_\psi^2)$$



Income Process, Volatility from Sabelhaus and Song (2010)

Back

$$\ln Y_{t+1} = f(t) + \ln P_{t+1} + \ln \theta_{t+1}$$
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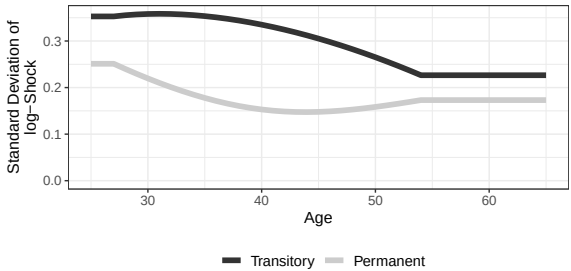


Frictions to Stock Holding

Back

Usual assumption is "per-period participation cost"

- Requires only one state variable for wealth.
- Induces selection and extensive margin adjustments.

My approach: there are two accounts

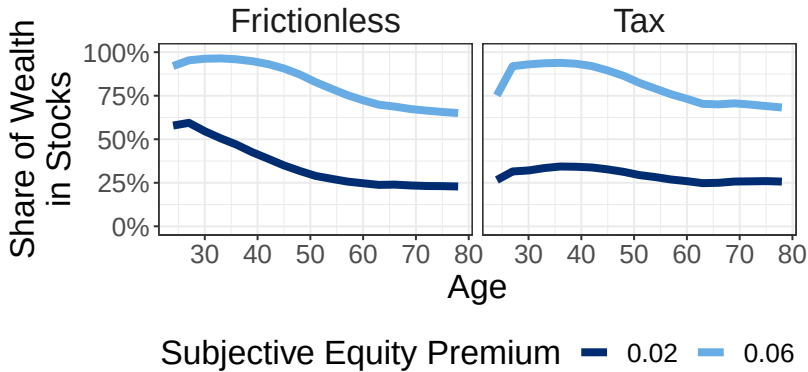
- Risk-free checking/saving account (M_t).
- Risky stock-market fund (N_t).

Frictions:

- Consumption must come from risk-free account.
- Risky to risk-free flows taxed at rate $\tau = 10\%$.

Frictions, Beliefs, and their Interaction

Back



Bellman equation

Back

$$V_t(M_t, N_t, P_t) = \max_{C_t, D_t} u(C_t) + \beta\delta_{t+1} E_t \left[V_{t+1} \left(M_{t+1}, N_{t+1}, P_{t+1} \right) \right]$$

Subject to:

$$-N_t \leq D_t \leq M_t, \quad 0 \leq C_t \leq \tilde{M}_t$$

$$\begin{aligned}
\tilde{M}_t &= M_t - D_t \left(1 - 1_{[D_t \leq 0]} \tau\right) \\
\tilde{N}_t &= N_t + D_t \\
A_t &= \tilde{M}_t - C_t \\
M_{t+1} &= R A_t + Y_{t+1} \\
N_{t+1} &= \tilde{R}_{t+1} \tilde{N}_t \\
P_{t+1} &= \Gamma_{t+1} \psi_{t+1} P_t \\
Y_{t+1} &= \theta_{t+1} P_{t+1}
\end{aligned}$$

Timing

Back

- Shocks $\{\tilde{R}, \psi_t, \theta_t\}$ are realized.
- Income and assets $\{P_t, Y_t, M_t, N_t\}$ are determined.
- Agent chooses D_t, rebalancing his assets.
- $\tilde{M}$ and $\tilde{N}$ are determined.
- Agent chooses consumption C_t out of his post-rebalancing liquid assets $\tilde{M}_t$.

Calibration Exercise - Loss Function

Back

"True" moments:

- I pool SCF waves from 1989 to 2019.
- Form 3-year age bins from $\{24, 25, 26\}$ to $\{78, 79, 80\}$.
- For each target moment, age-bin, and educational level
  - Find the relevant survey-weighted statistic.
- Stack all moments in target vector m_0.

Simulated counterparts: $\hat{m}(\vartheta)$.

Estimator:

$$\hat{\vartheta} = \arg\min_{\vartheta} \left(m_0 - \hat{m}\left(\vartheta\right)\right)' W \left(m_0 - \hat{m}\left(\vartheta\right)\right).$$

W re-scales errors as percentages of age-means of the true moments.