Financial Literacy, Human Capital and Economic Growth

Alberto Bucci* Riccardo Calcagno† Simone Marsiglio‡

Abstract

We analyze the relationship between financial literacy and economic growth by relying on an extended Uzawa-Lucas framework to account for the role of a financial sector that transfers savings intertemporally. The return generated by the financial sector depends on macroeconomic conditions and on the representative agent's degree of financial literacy. New financial literacy is produced by combining the existing stock of financial literacy with human capital, which needs to be endogenously allocated across production, accumulation of human capital, and the acquisition of financial literacy. We show that the presence of a financial sector changes the main characteristics of the balanced growth path (BGP) equilibrium with respect to the standard Uzawa-Lucas model, implying that the relative size of different economic sectors will change over time. We also identify two distinct ways through which finance can benefit economic growth, a "financial return channel" and a "human capital channel". Financial literacy has a positive impact on long-term economic growth if the financial sector return positively relates with the investment in financial literacy, or with its aggregate level.

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"...The recent crisis demonstrated the critical importance of financial literacy and good financial decision-making, both for the economic welfare of households and for the soundness and stability of the system as a whole. [...] The Federal Reserve recognizes that informed, educated consumers not only achieve better outcomes for themselves but, through careful shopping for and use of financial products, help to increase market efficiency and innovation." (Bernanke, 2011)

1 Introduction

The quick increase in the degree of financialization, coupled with the advent of new and more complex financial instruments that can be used to consume, save and invest have amplified the need for more financial literacy.[1] In parallel, following the global financial crisis of 2007-2008 and the recent events leading to COVID-19 pandemic crisis, the debate on the importance of financial literacy has gained even more momentum as the least knowledgeable investors are considered the most exposed to financial and economic

*Department of Economics, Business and Statistics, University of Milan, email: alberto.bucci@unimi.it.
†Department of Management and Production Engineering, Polytechnic University of Turin, email: riccardo.calcagno@polito.it.
‡Department of Economics and Management, University of Pisa, email: simone.marsiglio@unipi.it.

[1] For the sake of simplicity, in the present paper we shall use interchangeably such terms as financial literacy and financial knowledge. For a more systematic review of the possible definitions of financial literacy and of its possible synonyms in the literature, see Huston (2010) and more recently Goyal and Kumar (2021, pp. 80-81).

crunches.[2] Moreover, financial market liberalizations and policy reforms aimed at promoting retirement savings through private pension funds and individual retirement accounts have caused an ongoing shift in decision-making responsibility away from governments, institutions and employers towards private individuals. The importance of a sufficient level of financial literacy for individuals and households to take wiser decisions in various domains is by now widely recognized.[3]

While the microeconomic literature has already reached a wide consensus on the importance of financial literacy, so far fewer contributions have investigated the possible macroeconomic effects of a population becoming more financially literate. In particular, to our knowledge there is no thorough theoretical work analyzing, within a dynamic setting, the impact that financial literacy can have on equilibrium economic growth in the very long-run. The present paper aims at filling this important gap in the existing macroeconomic literature.

At the macroeconomic level, existing analyses of the impact of financial literacy generally do not look at its effects on long-run economic growth.[4] For instance, Ameriks et al. (2003), Lusardi and Mitchell (2007), and Jappelli and Padula (2013) provide evidence of a positive link between financial literacy, saving decisions and wealth accumulation. According to Brown et al. (2016), those being more financially literate better understand new and complex financial products, and hence can more effectively protect themselves against longevity risk in retirement. By using a stochastic life-cycle model that endogenizes the decision to acquire financial knowledge, Lusardi et al. (2017) show that gaps in financial literacy amplify differences in wealth accumulation patterns and the consequent perpetuation in wealth inequality. More specifically, they find that 30-40% of US wealth-inequality can potentially be attributed to disparities in financial knowledge in the population. Along the same lines, Lo Prete (2013, 2018) tests how the ability to take advantage of different financial opportunities (measured by financial knowledge) may help reducing inequality across countries and over time.

Our paper aims to contribute to the theoretical debate on the macro-effects of financial literacy by analyzing the impact of this variable on long-run equilibrium economic growth. To do so, we extend the path-breaking Uzawa (1965) and Lucas (1988)'s two-sector human capital-based endogenous growth model with the addition of a financial sector that transfers savings intertemporally. We also provide the agents with the possibility to invest in financial literacy, to be interpreted as knowledge specific to the financial sector. This sector' efficiency depends on the macroeconomic conditions of the economy, and is therefore determined endogenously.

It is worth mentioning at this stage that, while financial literacy and human capital are not synonyms, they are strongly related to each other (Arrondel, 2018). Individuals can make use of relatively basic economic or mathematical concepts generally learned during their years of study to take appropriate financial decisions. With this in mind, our model builds on the idea that, like human capital, financial literacy is an

[2]See, for example, Gerardi et al. (2010), Hasler et al. (2018), Brown et al. (2016a), Guiso and Viviano (2015). Feng et al. (2019), using a Bayesian two-part latent variable modelling approach, have identified the simultaneous impact of financial literacy on household debt and assets. They show that households with insufficient financial knowledge are more financially vulnerable because they are more likely both to have fewer assets and to choose high-cost unsecured debts that expose them more to potential financial constraints. By contrast, financial knowledge is found to enable individuals, among other things, to plan for wealth accumulation (Ameriks et al. 2003), to be more financially included (Grohmann et al. 2018), and to choose investments that are the most suitable for their own needs (Bianchi 2018).

[3]Several papers clearly document a positive correlation between (different measures of) financial literacy and better financial decisions. For example, people with higher levels of financial knowledge are more likely to participate in financial markets and to invest in stocks (Christelis et al. 2010; Yoong 2011; Van Rooij et al. 2011); have both better diversified portfolios (Guiso and Jappelli 2008; Von Gaudecker 2015), and portfolios yielding higher returns (Calvet et al., 2009); earn higher yields on deposit accounts (Deuflhard et al. 2019). Hsiao and Tsai (2018) provide evidence of a positive impact of financial knowledge on trading in leveraged derivative products, an important means of hedging financial risks in portfolios. Financial literates better perform in peer-to-peer lending markets (Chen et al. 2018) and choose mutual funds with lower fees (Hastings and Tejeda-Ashton, 2008).

[4]A notable exception is represented by Greenwood and Jovanovic (1990), who were among the first to demonstrate that financial institutions that encourage financial knowledge (by producing and diffusing, for example, better information on firms) induce a more efficient allocation of capital investment, and therefore promote both financial development and economic growth in the long-run.

intangible asset that forward-looking agents can accumulate over time. This explains why a macroeconomic theory of financial literacy needs to be embedded within the Uzawa-Lucas two-sector endogenous growth framework.

Two main assumptions characterize more precisely our model. The first is that financial literacy requires two inputs to be augmented: education – or human capital – and the stock of specialized financial knowledge already existing.[5] We ground this hypothesis on the fact that several studies (e.g. Lusardi and Mitchell 2008; Cole et al., 2011) have already demonstrated that individuals with higher levels of education are the most likely to be financially literate. Moreover, investigations on the effect of human capital (education) on stock market participation show that college-educated are more likely to own stocks (Haliassos and Bertaut 1995; Campbell, 2006; Lusardi and de Bassa Scheresberg 2013). Cole and Shastry (2008) argue that one more year of schooling increases the probability of financial market participation by 7-8%. This result is consistent with the findings of Thomas and Spataro (2018), according to whom an increase in the number of years of schooling raises the probability of investing in the stock market through externalities or peer–effects. Looking a step further, empirical analyses on stock holding reveal that including controls for educational attainment does enhance the significance of the variable financial literacy (Van Rooij et al., 2011; Behrman et al., 2012; Lusardi and de Bassa Scheresberg, 2013). These findings emphasize the idea that general knowledge (education) and specialized knowledge (financial literacy) both contribute positively to effective financial decision-making. Mandell (2008) and Al-Bahrani et al. (2020) have demonstrated that the correlation between financial literacy and education is present at the early stages of the life-cycle, and that financial information is especially related to numeracy skills. Last but not least, the belief that financial knowledge may be self-productive (in the sense that greater initial financial knowledge contributes to enhance the efficiency with which new financial knowledge is obtained over time) has been suggested, among the first, by Delavande et al. (2008).[6] Together, all these pieces of evidence clearly show that both the existing amount of financial knowledge and the number of man-hours purposely devoted to the production of new financial literacy in the form of human capital are two distinctive factors in the process of raising the degree of financial knowledge of a population.

The second key assumption of our model hinges on the idea that, at the macroeconomic level, the return on agents' savings may be influenced not only by the amount of their investment in financial literacy (as in Arrow, 1987; Jappelli and Padula, 2013; Lusardi et al., 2017) but also by their investment in human capital. To justify this hypothesis, it is worth mentioning here a recent work by Kim et al. (2016) who try to explain the tendency to maintain one's investment portfolio for very long periods of time, the so-called "investor inertia". Such phenomenon, by reducing the return on savings, is now universally interpreted as evidence of financial illiteracy among people. The authors build a life-cycle dynamic model where the time required to manage one's financial portfolio is traded-off with the time devoted to developing job-specific human capital. Following Arrow (1962) and Becker (1964), they posit that job-specific human capital is, in turn, accumulated through learning by doing. Using reasonable parameter values, their model is able to reproduce the same patterns of portfolio inertia across age groups suggested by existing empirical evidence. Two relevant features make our paper especially different from Kim et al. (2016)'s contribution. The first difference rests in the aim of the contribution. Whereas Kim et al. (2016) build their dynamic model (where human capital and financial literacy are introduced as two different endogenous forms of investment) with the objective of explaining investor inertia, our purpose is to shed light on the theoretical nexus between financial knowledge and education (human capital) and its role in eventually affecting the equilibrium economic growth rate of a country in the very long-run. As already said, to the best of our knowledge, we are the first to do this. The

[5]The OECD (2020) defines financial education as *"...The process by which financial consumers/investors improve their understanding of financial products, concepts and risks and, through information, instruction and/or objective advice, develop the skills and confidence to become more aware of financial risks and opportunities, to make informed choices, to know where to go for help, and to take other effective actions to improve their financial wellbeing."*

[6]Delavande et al. (2008, Eq. 4, p. 10) estimate a production function for financial knowledge using data from the US Cognitive Economic Survey.

second major difference has to do with the modelling of the process of investment in both human capital and financial literacy. Whereas Kim et al. (2016)'s notion of human capital is informed by the idea of job-specific skills accumulated by working (as in Becker, 1964), in our paper human capital is not accumulated on the job, but rather by purposefully investing time (i.e., human capital in terms of man-hours) in this economic activity, which competes with other economic activities, (in particular with the accumulation of new financial knowledge and with the use of human capital for production purposes, as in Uzawa-Lucas) for the same scarce input (time). We do believe this is another significant novel feature of our contribution with respect to the existing literature. Thanks to this feature, our model emphasizes that, from a macroeconomic perspective, the investment in financial literacy ultimately gives rise to a dynamic cost/benefit trade-off. On the benefits side, a better level of financial knowledge permits households to obtain higher returns on their savings and/or asset holdings (as in Lusardi et al., 2017; Arrow, 1987; Jappelli and Padula 2013). On the cost side, the investment in financial literacy increases the opportunity cost of time for human capital accumulation (as in Kim et al., 2013). The consequences of this trade-off are crucial in our model and drive its main results.

Our analysis leads to the following findings. The first and most prominent one is that embedding a financial sector that transfers savings intertemporally in an otherwise standard Uzawa-Lucas model dramatically changes the main characteristics of this last setup. More in detail, the presence of a financial sector changes the long-term growth rate of the model-economy and the main characteristics of its balanced growth path (BGP) equilibrium with respect to Uzawa-Lucas. This implies that over time the relative size of different economic sectors will change, with physical capital growing faster than production and consumption, which in turn grow faster than human capital and the stock of financial literacy.

Moreover, we identify two distinct ways through which finance can benefit economic growth. The first operates through a change in the efficiency of the financial sector over time, and we define it as the "financial return channel". The second, the "human capital channel", works through a change in the rate at which human capital is ultimately accumulated. The "financial return" channel can be described as follows. The higher opportunity cost of time for human capital accumulation in the presence of a financial sector leads to a lower growth rate of human capital relatively to an analogous economy without the financial sector. Coeteris paribus, this lowers the rate of economic growth. At the same time, when the financial sector return is highly sensitive to the acquisition of new financial literacy, agents accumulate financial literacy and by doing so they can earn higher financial returns. If the financial sector is particularly effective in intertemporally transferring savings, this second effect more than compensates for the first diversion effect, leading the economic growth rate to exceed the one in a standard Uzawa-Lucas model. Alternatively, the "human capital" channel is relevant when the financial sector return depends very strongly on human capital, so that it becomes optimal to devote more time to general education than in the model without finance. The higher growth rate of human capital promotes economic growth.

Finally, we highlight the role of financial literacy in spurring long run economic growth. Acquiring new financial literacy per se does not amplify growth. Financial literacy has a positive impact on long-term growth if the return generated by the financial sector positively relates with the investment in financial literacy, or with its aggregate level.

The paper is structured as follows. In section 2 we introduce our endogenous growth framework. In section 3 we illustrate and discuss the main results of the analysis. In doing so, we shall focus on the long-run BGP equilibrium properties of the model and its growth rate. Section 4 compares the growth rate of our economy with the one in a standard Uzawa-Lucas model without a financial sector. Section 5 concludes and proposes possible paths for future research.

2 The Model

We analyze a discrete-time endogenous growth model à-la Uzawa-Lucas (1988) extended for a financial sector. For the sake of simplicity, we abstract from population growth and capital depreciation.

The social planner seeks to maximize social welfare subject to the evolution of physical capital k_t, human capital h_t and financial literacy a_t by determining consumption $c_t > 0$ and the shares of human capital to employ in the production of the final consumption good, $0 < u_t < 1$ and in the accumulation of financial literacy, $0 \leq \nu_t < 1$.[7] Social welfare is the infinite discounted sum of the instantaneous utilities, where $\beta \in (0, 1)$ denotes the discount factor. The representative agent's instantaneous utility function depends only on consumption and takes a logarithmic form, $u(c_t) = \ln(c_t)$.

The unique final consumption good is produced by employing physical capital and a share of human capital u_t determined endogenously, through a Cobb-Douglas technology:

$$y_t = k_t^{\alpha}(u_t h_t)^{1-\alpha} \tag{1}$$

where $0 < \alpha < 1$ is the physical capital share, and output can be either consumed or saved and invested in physical capital accumulation.

Human capital formation depends on the remaining level of human capital not allocated to production of either the consumable good or financial literacy. New human capital is produced through a linear technology determining the accumulation of human capital as follows:

$$h_{t+1} = b(1 - u_t - \nu_t)h_t \tag{2}$$

where $b > 1$ measures the productivity of education in human capital formation.

While human capital quantifies general skills (i.e., education, cognitive ability and numeracy skills), financial literacy measures knowledge and skills specific to the financial sector. Similarly to Kim et al. (2013), human capital plays an essential role in accumulating financial knowledge.[8] Accordingly, we assume that financial literacy is produced by combining existing financial literacy and a share of human capital decided endogenously ν_t. New financial knowledge is produced through a Cobb-Douglas technology (Delavande et al., 2008), determining the accumulation of financial literacy, as follows:

$$a_{t+1} = (\nu_t h_t)^{1-\xi} a_t^{\xi} \tag{3}$$

with $0 < \xi < 1$ measuring the elasticity of financial literacy production with respect to its existing stock. The acquisition of new financial literacy therefore combines its existing stock with the allocated share of human capital, $\nu_t h_t$.[9] Different from human capital, financial literacy is not an input in the production of the final good and thus it has no effect on output (see (1)).

Physical capital accumulation depends on saving augmented for the efficiency of the financial sector. The efficiency of the financial sector depends on macroeconomic conditions and in particular on income, human capital, and financial literacy, both in terms of stock and investment flow. The financial sector transfers intertemporally savings, and its efficiency determines how many units $R > 0$ of saving are transferred to time $t + 1$ for every unit of saving at time t. There exist several channels through which the financial sector

[7] Few works have developed similar three-sector extensions of the Uzawa-Lucas model to account for the accumulation of knowledge (La Torre and Marsiglio, 2010; La Torre et al., 2015). Different from them, in our setting the third sector produces an output (i.e., financial literacy) which does not represent an input in the production of the consumption good.

[8] As in Kim et al. (2013), also in our model an activity pursuing personal financial wellbeing (i.e. portfolio management in their framework, the acquisition of financial knowledge in ours) costs time and effort otherwise dedicated to a productive activity (i.e. the accumulation of job-specific human capital via learning by doing in their model, the accumulation of productive human capital in ours).

[9] We abstract from other costly inputs potentially relevant in reality, such as professional advice, educational courses, books and magazines.

affects the aggregate return on savings. Financial intermediaries are able to better funnel savings to firms (Pagano, 1993), allow agents to create diversified portfolios with higher expected returns (King and Levine, 1993), improve the access to education increasing the returns on investment (De Gregorio, 1996). All these theoretical explanations suggest that R could be higher than one, at least for some values $(k_t, h_t, a_t, u_t, \nu_t)$. At the same time, financial intermediaries drain resources away from physical capital accumulation, thereby reducing R. The return on savings may also depend on the aggregate level of financial literacy (a_t) and on the amount of financial knowledge accumulated during period t (i.e. $\nu_t h_t$) because of the insight of Arrow (1987) (see also Lusardi et al., 2017, and Jappelli and Padula, 2013). Investors with higher degree of financial literacy are able to select better investment opportunities, and thereby obtain a higher return on their savings. Moreover, it has been shown empirically (Campbell, 2006, van Rooij et al. 2011) that financial literacy is positively associated to stock market participation. Since the stock market typically pays a premium over risk-free investment, higher stock market participation relates to higher return on savings. These findings suggest a positive relationship between knowledge specific to finance and the return on savings. At a macroeconomic level, the financial system allocates aggregated saving to the productive sector. If financial intermediaries improve the allocation of capital (Greenwood and Jovanovic, 1990), and increase the efficiency of investment (Bencivenga and Smith, 1991), a higher return on savings originates higher return on capital invested. For the moment, we do not take a stand concerning the drivers of the financial sector efficiency, and we allow R to depend on output y_t and on financial literacy, both in the form of stock a_t and flow ν_t. Accordingly, we write $R(y_t, a_t, \nu_t)$. It follows that the dynamic evolution of physical capital reads as:

$$k_{t+1} = R(y_t, a_t, \nu_t)(y_t - c_t),$$

which, from (1), can be also written as

$$k_{t+1} = R(k_t, h_t, a_t, \nu_t, u_t)(y_t - c_t) \tag{4}$$

We suppose that the financial sector gross return function R takes a standard neoclassical form, that is it is everywhere continuous and differentiable, strictly increasing and concave in all its arguments. Note that if $R = 1$ for all vectors $(k_t, h_t, a_t, \nu_t, u_t)$, then our model reduces to a standard three-sector Uzawa-Lucas model (see for example La Torre et al., 2015). Equations (1), (2) and (4) show that the allocation of a share ν_t of human capital into financial knowledge involves a trade-off. On the benefits side, if R is positively related to ν_t, higher ν_t increases the return on capital invested and therefore increases the future stock of physical capital. On the cost side, higher ν_t may reduce the amount of human capital that can be devoted to production, u_t, and to the acquisition of new human capital, $1 - u_t - \nu_t$.

Given the initial conditions k_0, h_0 and a_0, the social planner's problem can be summarized as follows:

$$\begin{aligned} \max_{\{c_t, u_t, \nu_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \ln(c_t) \\ s.t. \quad & k_{t+1} = R(k_t, h_t, a_t, u_t, \nu_t)(y_t - c_t) \\ & h_{t+1} = b(1 - u_t - \nu_t)h_t \\ & a_{t+1} = (\nu_t h_t)^{1-\xi} a_t^{\xi} \\ & k_0, h_0, a_0 > 0 \end{aligned} \tag{5}$$

The above problem shows that the financial sector and financial literacy potentially play a crucial role in determining macroeconomic outcomes. In particular, the efficiency of the financial sector affects physical capital accumulation, and therefore production and consumption at equilibrium. The accumulation of financial literacy affects the dynamics of human capital, which is key to the determination of the long-run growth rate of the economy, as in the standard Uzawa-Lucas model.

3 The Long Run Equilibrium

The next proposition obtains a closed form solution of problem (5).

Proposition 1. *Let $\varepsilon_{R,i} \geq 0$ with $i = \{k, h, a, u, \nu\}$ denote the elasticity of the financial sector's return function R with respect to variable i. If $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ then:*

1. The optimal policy rules for consumption c_t, for the share of human capital allocated respectively to the production of final good u_t and of new financial literacy ν_t are given by:

$$c_t = \frac{1-\alpha\beta-\beta\varepsilon_{R,k}}{1-\beta\varepsilon_{R,k}} k_t^{\alpha} \overline{u}^{1-\alpha} {h_t}^{1-\alpha} \tag{6}$$

$$u_t = \overline{u} = \frac{1-\beta\Theta}{\Delta} \in (0,1) \tag{7}$$

$$\nu_t = \overline{\nu} = \frac{1-\beta\Theta}{\Delta} \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \in [0,1) \tag{8}$$

where

$$\Theta = \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{9}$$

$$\Delta = \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{10}$$

The optimal share $\overline{u}$ increases with $\varepsilon_{R,u}$ and decreases with $\varepsilon_{R,h}$, while $\overline{\nu}$ increases with $\varepsilon_{R,\nu}$ and decreases with $\varepsilon_{R,h}$.

2. The optimal dynamics of physical capital k_{t+1}, human capital h_{t+1} and financial literacy a_{t+1} are:

$$k_{t+1} = R^{\frac{\alpha\beta}{1-\beta\varepsilon_{R,k}}} k_t^{\alpha} \overline{u}^{1-\alpha} {h_t}^{1-\alpha} \tag{11}$$

$$h_{t+1} = b(1-\overline{u}-\overline{\nu})h_t \tag{12}$$

$$a_{t+1} = \overline{\nu}^{1-\xi} h_t^{1-\xi} a_t^{\xi} \tag{13}$$

Proposition 1 derives explicitly the optimal policy and the optimal dynamics of physical capital, human capital and financial literacy. For these dynamics to have a meaningful economic interpretation the elasticity of the financial sector's return with respect to physical capital $\varepsilon_{R,k}$ needs to be sufficiently small,[10] while no restrictions on the magnitude of other elasticities are required.[11] The size of these elasticities plays a critical role in determining the optimal policy rules.

If the financial sector intertemporally transfers savings into physical capital one-to-one, then $R = 1$ for all possible values of k, h, a, u, ν, and all elasticities $\varepsilon_{R,i}$ are equal to zero. The dynamics in Proposition 1 reduce to those in a standard Uzawa-Lucas model (see for example La Torre et al., 2015): the share of human capital devoted to production equals $\overline{u} = 1-\beta$, and consumption c_t represents a fixed quota of production, $c_t = (1-\alpha\beta)y_t$. When $R = 1$ it is optimal not to invest in financial literacy ($\overline{\nu} = 0$). Financial literacy does not contribute to the production of the final good, as in (1) and therefore it generates no economic benefits. However, the acquisition of new financial literacy has a cost in so far it reduces the amount of human capital that can be allocated to production and to the accumulation of new human capital.

If $R \neq 1$ for some values of k, h, a, u, and ν then the financial sector and financial literacy affect the optimal policy rules and the macroeconomic outcomes both in the short run and in the long run. Some specific cases are interesting.

[10] If the elasticity $\varepsilon_{R,k} > \frac{1-\alpha\beta}{\beta}$ the solution of the model exhibits negative consumption. The condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta} < \frac{1-\alpha\beta}{\beta}$ is sufficient to ensure that $\overline{u} > 0$, $\overline{\nu} \geq 0$ and that $\overline{u}+\overline{\nu} < 1$ (see the proof in the Appendix for the details).

[11] Recall that we have assumed the return function R takes a neoclassical form, so that all the elasticities $\varepsilon_{R,i}$ are constant and non-negative.

If $\varepsilon_{R,a} = \varepsilon_{R,\nu} = 0$ then financial literacy does not affect the financial sector return and $\overline{\nu} = 0$. When $\varepsilon_{R,h} > \varepsilon_{R,u}$ (respectively, $\varepsilon_{R,h} < \varepsilon_{R,u}$), we have that $\Theta > 1$ (resp. $\Theta < 1$) and $\overline{u} < 1-\beta$ (resp. $\overline{u} > 1-\beta$). If the return generated by the financial sector is relatively more sensitive to the existing stock of human capital than to the share of human capital devoted to production, then it is optimal to accumulate more human capital than in the standard Uzawa-Lucas model (and viceversa). The financial sector provides additional incentives to invest in general education, because a higher level of general education produces higher return on savings on the financial markets.

Another particular case worth discussing is $\varepsilon_{R,u} + \varepsilon_{R,\nu} = \varepsilon_{R,h}$. Under this condition we obtain $\Theta = 1$ from (9), and $\overline{u}+\overline{\nu} = 1-\beta$.[12] Therefore, (see (2)), the growth rate of human capital equals $b(1-\beta)$, as in the standard Uzawa-Lucas setup and the presence of a financial sector does not affect the dynamics of human capital. This result holds irrespectively of the effect of financial literacy on the financial sector productivity, i.e. for any $\varepsilon_{R,a} \geq 0$ and $\varepsilon_{R,\overline{\nu}} \geq 0$. The special case $\varepsilon_{R,u} + \varepsilon_{R,\nu} = \varepsilon_{R,h}$ allows us to clarify the role of the existing level of financial literacy on the optimal allocation of resources. If $\varepsilon_{R,u} + \varepsilon_{R,\nu} = \varepsilon_{R,h}$ and $\varepsilon_{R,a} > 0$ it is optimal to allocate a quota of the existing human capital to the production of new financial literacy ($\overline{\nu} > 0$), because higher knowledge specific to finance makes the financial sector more efficient. The quota of human capital devoted to production reduces correspondingly, so that the overall dynamics of human capital stays as in the standard Uzawa-Lucas model.

The comparative statics of the optimal allocation of human capital among production ($\overline{u}$), creation of new financial knowledge ($\overline{\nu}$) and new human capital ($1-\overline{u}-\overline{\nu}$) are not always univocal. The unambiguous results reported in Proposition 1 have a natural interpretation. It is optimal to allocate a larger share of human capital to a certain activity, for example to the investment in new financial literacy, when the elasticity of the financial return function w.r. to such activity increases. Following the same logic, if the return on saving becomes relatively more sensitive to the existing stock of human capital ($\varepsilon_{R,h}$ increases) then it is optimal to accumulate relatively more human capital (the sum $\overline{u}+\overline{\nu}$ decreases).

We now characterize the long run steady state outcome of our economy, which is represented by a balanced growth path (BGP) equilibrium.

Proposition 2. *Assume that* $b > \frac{1}{\beta\Theta'}$ *and* $\varepsilon_{R,k} = 0$, *where*

$$\Theta' = \frac{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{14}$$

Then along the BGP the shares of human capital allocated to production and to financial literacy accumulation are constant,

$$\overline{u} = \frac{1-\beta\Theta'}{\Delta'} \in (0,1) \tag{15}$$

$$\overline{\nu} = \frac{1-\beta\Theta'}{\Delta'}\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}} \in [0,1) \tag{16}$$

where

$$\Delta' = \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}} \tag{17}$$

while the growth rates of consumption (γ_c), *physical capital* (γ_k), *human capital* (γ_h), *financial literacy* (γ_a)

[12] See the proof of Proposition 1 for the latter equality.

and production (γ_y) are constant and given by:

$$\gamma_h = \gamma_a = b\beta\Theta' - 1 > 0 \tag{18}$$

$$\gamma_k = (1+\gamma_R)^{\frac{1}{1-\alpha}}(1+\gamma_h) - 1 > 0 \tag{19}$$

$$\gamma_y = \gamma_c = (1+\gamma_R)^{\frac{\alpha}{1-\alpha}}(1+\gamma_h) - 1 > 0, \tag{20}$$

where $\gamma_R \geq 0$ is the growth rate of the financial sector return.

The growth rate of human capital γ_h increases with $\varepsilon_{R,h}$ and decreases with $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$, while the effect of $\varepsilon_{R,a}$ on γ_h is ambiguous: γ_h increases (decreases) with $\varepsilon_{R,a}$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} > 0$ (< 0), while γ_h does not depend on $\varepsilon_{R,a}$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} = 0$.

Provided that the elasticity of the financial sector return with respect to physical capital is null (i.e., $\varepsilon_{R,k} = 0$), Proposition 2 explicitly determines the growth rate of the main variables along the BGP equilibrium. If the productivity of human capital in the creation of new human capital is large enough (i.e., $b > \frac{1}{\beta\Theta'}$), then all state variables grow at a strictly positive and constant rate. The human capital allocation variables u_t and ν_t instead are constant through time, as in Proposition 1. It is interesting to note that whenever the financial sector return is constant (i.e., $R = 1$ and $\gamma_R = 0$) along the BGP physical and human capital, consumption, production and financial literacy all grow at the same common rate, given by (18). If instead the financial sector return grows at a strictly positive rate, this introduces a wedge between the growth rates of the main variables of the model. Specifically, physical capital grows faster than production and consumption, which in turn grow faster than human capital and the stock of financial literacy. The fact that variables grow at different rates imply that over time the relative size of different economic sectors will change.

The comparative statics of the growth rate of human capital with respect to the financial sector return elasticities are intuitive. The higher the elasticity of the financial return to the existing stock of human capital $\varepsilon_{R,h}$, the higher the incentive to invest in new human capital, and the higher γ_h. Conversely, when the elasticities $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$ increase it becomes optimal to devote a larger share of human capital in production and in the accumulation of new financial knowledge rather than in general knowledge so that γ_h decreases. If $\varepsilon_{R,u} + \varepsilon_{R,\nu} = \varepsilon_{R,h}$, then $\Theta' = 1$ and the stock of human capital grows at the same rate as in a standard Uzawa-Lucas model without the financial sector, irrespectively of the size of the elasticity $\varepsilon_{R,a}$.

So far we have not been able to determine explicitly the growth rate of our economy, γ_y as this critically depends on γ_R, the growth rate of the financial sector return. In order to do so and to understand more clearly the determinants of long-term growth, we consider the following functional form for R:

$$\begin{aligned} R_t &= (u_t h_t)^{\varepsilon_u}(\nu_t h_t)^{\varepsilon_\nu} h_t^{\chi} a_t^{\varepsilon_a} \\ &= \overline{u}^{\varepsilon_u}\overline{\nu}^{\varepsilon_\nu} h_t^{\varepsilon_u+\varepsilon_\nu+\chi} a_t^{\varepsilon_a} \end{aligned} \tag{21}$$

where $\varepsilon_i \geq 0$ denote the elasticities of R with respect to $i = \{u, \nu, a\}$ respectively, and $\varepsilon_h = \varepsilon_u + \varepsilon_\nu + \chi$. By Proposition 2, along the BGP we have:

$$\frac{R_t}{R_{t-1}} = 1 + \gamma_R = \frac{\overline{\nu}^{\varepsilon_\nu}\overline{u}^{\varepsilon_u} h_t^{\varepsilon_h} a_t^{\varepsilon_a}}{\overline{\nu}^{\varepsilon_\nu}\overline{u}^{\varepsilon_u} h_{t-1}^{\varepsilon_h} a_{t-1}^{\varepsilon_a}} = \left(\frac{h_t}{h_{t-1}}\right)^{\varepsilon_h}\left(\frac{a_t}{a_{t-1}}\right)^{\varepsilon_a} = (1+\gamma_h)^{\varepsilon_h+\varepsilon_a}$$

because $\gamma_a = \gamma_h = b\beta\Theta' - 1$ (see (18)). From (20), the growth rate of output is given by:

$$1 + \gamma_y = \left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}}$$

with Θ' as in (14). Given that $b > \frac{1}{\beta\Theta'}$, we have that γ_y exceeds γ_h. The presence of a financial sector makes the output grow at a rate which is higher than the growth rate of human capital. This because the financial

industry generates a positive return by the intertemporal allocation of savings.[13]

Moreover,

$$1+\gamma_y = \left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}} = \exp\left\{\ln\left(\left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}}\right)\right\} = \exp\left\{\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\ln\left(b\beta\Theta'\right)\right\}$$

$$\begin{aligned}
\frac{\partial\gamma_y}{\partial\varepsilon_h} &= \exp\left\{\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\ln\left(b\beta\Theta'\right)\right\}\frac{\partial\left[\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\ln(b\beta\Theta')\right]}{\partial\varepsilon_h} \\
&= \left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\ln\left(b\beta\Theta'\right)+\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\frac{1}{b\beta\Theta'}b\beta\frac{\partial\Theta'}{\partial\varepsilon_h}\right) \\
&= \left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\ln\left(b\beta\Theta'\right)+\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\frac{\frac{\partial\Theta'}{\partial\varepsilon_h}}{\Theta'}\right) > 0
\end{aligned}$$

because $b > \frac{1}{\beta\Theta}$ and $\frac{\partial\Theta'}{\partial\varepsilon_h} > 0$. Analogously,

$$\begin{aligned}
\frac{\partial\gamma_y}{\partial\varepsilon_a} &= \exp\left\{\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\ln\left(b\beta\Theta'\right)\right\}\frac{\partial\left[\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\ln(b\beta\Theta')\right]}{\partial\varepsilon_a} \\
&= \left(b\beta\Theta'\right)^{1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\ln\left(b\beta\Theta'\right)+\left(1+\frac{\alpha(\varepsilon_h+\varepsilon_a)}{1-\alpha}\right)\frac{\frac{\partial\Theta'}{\partial\varepsilon_a}}{\Theta'}\right)
\end{aligned}$$

whose sign depends on the sign of $\frac{\partial\Theta'}{\partial\varepsilon_a}$. If $\varepsilon_{R,u}+\varepsilon_{R,\nu}=\varepsilon_{R,h}$, then $\frac{\partial\Theta'}{\partial\varepsilon_a}=0$ and γ_y increases with ε_a. In other words, in the case $\varepsilon_{R,u}+\varepsilon_{R,\nu}=\varepsilon_{R,h}$ a higher elasticity ε_a makes it optimal to devote more human capital to the creation of new financial knowledge, and to allocate less human capital to production. The level of financial literacy therefore grows more rapidly, and γ_R increases. Ultimately (see (20)), also the rate of economic growth γ_y increases.

4 The Role of the Financial Sector and of Financial Literacy

We now analyze the role of the financial sector and of financial literacy in determining long run macroeconomic outcomes. In particular, we compare our model results which those arising in a traditional Lucas-Uzawa framework without a financial sector. In order to make a meaningful comparison we consider that all structural parameters are the same across the two models, with the exception of those related to financial activities.

In a traditional Lucas-Uzawa setup R always equals one, so that all elasticities $\varepsilon_{R,i}$ are null as well as the share of human capital devoted to financial literacy accumulation. Along the BGP state all variables grow at the same constant rate γ^{UL} and the share of human capital allotted to production $\overline{u}^{UL}$ is constant (Lucas 1988):

$$\gamma^{UL} = b\beta - 1 \tag{22}$$

$$\overline{u}^{UL} = 1-\beta, \tag{23}$$

The BGP is well defined, i.e., $\gamma^{UL}>0$ and $0<\overline{u}^{UL}<1$, whenever $b>\frac{1}{\beta}$ (see La Torre et al., 2015). Note that such a condition does not necessarily ensure sustained growth in our financial-sector-extended model (see (18) and (20)). Therefore, in the following we shall restrict our analysis to the parameter region $b>\max\{\frac{1}{\beta\Theta'},\frac{1}{\beta}\}$. Under this condition, both frameworks are characterized by endogenous growth and their

[13] Notice that the difference $\gamma_y-\gamma_h$ increases with α, the elasticity of output to physical capital.The financial sector pushes economic growth through the increased accumulation of physical capital, by converting savings into investments.

BGP is well defined. For easiness of comparison we also assume that the productivity of education in human capital formation b is the same across the two setups. The next proposition compares the growth rates in our framework and in a traditional Uzawa-Lucas one.

Proposition 3. *Consider an Uzawa-Lucas framework with γ^{UL} as in (22) and $\overline{u}^{UL}$ as in (23), and our model described in (5). Assume that $b > \max\{\frac{1}{\beta\Theta'}, \frac{1}{\beta}\}$ is the same in the two setups. Then along the BGP the following results hold:*

(i) If $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\gamma_h \leq \gamma^{UL}$ and $\gamma_y \geq \gamma^{UL}$ provided that $1 + \gamma_R \geq (\frac{1}{\Theta'})^{\frac{1-\alpha}{\alpha}}$;

(ii) If $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ then $\gamma_h > \gamma^{UL}$ and $\gamma_y > \gamma^{UL}$.

Proposition 3 states that the size of the elasticities of the financial sector's return with respect to the human capital stock and its allocation determines whether finance can benefit economic growth. Specifically, it identifies two distinct channels through which this may occur, described respectively by points (i) and (ii) in Proposition 3.

When $\varepsilon_{R,u}+\varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ (point (i)) the beneficial effect of finance on growth occurs through the "financial return channel". In this case the financial sector return depends more strongly on financial literacy formation (and production efforts) than on the accumulation of human capital. This makes convenient to devote less time to education, relatively to an analogous economy without the financial sector, leading to a lower growth rate of human capital γ_h and potentially to a lower rate of growth γ_y. However, if the financial sector is particularly effective in intertemporally transferring savings, it can more than compensate for such financial-literacy-related diversion effect, leading the economic growth rate to exceed the one in the standard Uzawa-Lucas model. More precisely, if the growth rate of the financial sector return is sufficiently high (i.e., $1 + \gamma_R \geq (\frac{1}{\Theta'})^{\frac{1-\alpha}{\alpha}} > 1$), then the increase in effectiveness of the intertemporal transfer of saving may more than compensate for the detrimental effects induced by the lower human capital accumulation. If this is the case the growth rate of the economy exceeds the one that would be achieved without finance.

Instead, when $\varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h}$ (point (ii)) the presence of a financial sector, together with the possibility to accumulate financial literacy, generate beneficial effects on growth through the "human capital channel". In this case the financial sector's return depends very strongly on human capital so that it is convenient to devote more time to education than in the model without finance. This in turn leads to a growth rate of human capital higher than in the standard Uzawa-Lucas setup. The high(er) rate of human capital growth promotes economic growth, independently of the accumulation of financial literacy.

In order to get a better sense of these two effects and to clarify the role played by financial literacy, let us consider a simplified return function for the financial sector. In particular, suppose that $\varepsilon_{R,u} = 0$ and $\varepsilon_{R,a} = 0$, so that R depends only on ν and h.[14]

If $\varepsilon_{R,\nu} \geq \varepsilon_{R,h} > 0$, from (16) we have that $\overline{\nu} > 0$. Compared to the standard Uzawa-Lucas setup, the presence of a financial sector lowers the share of human capital devoted to production and induces to invest in financial literacy. The investment in financial literacy exactly offsets the reduction in $\overline{u}$ when $\varepsilon_{R,\nu} = \varepsilon_{R,h}$.[15] If $\varepsilon_{R,\nu} > \varepsilon_{R,h}$ the investment in financial literacy diverts resources away from human capital formation.[16] As the human capital stock grows in every period ($1 - \overline{u} - \overline{\nu} > 0$), also the financial sector return increases, $\gamma_R > 0$. If the rate of growth γ_R is sufficiently high, then the additional resources generated by the financial sector more than compensate the negative effect due to a lower human capital growth. In this case, the growth rate of the economy is higher than in Uzawa-Lucas. The investment in financial literacy is the key driver of higher economic growth because it increases the return generated by financial sector (the "financial return" channel).

If $\varepsilon_{R,\nu} < \varepsilon_{R,h}$, then human capital grows at a higher rate than in the otherwise identical model without financial sector (by Proposition 3 (ii)). In this case the share of human capital devoted to education

[14] Recall that the results in Proposition 2 and Proposition 3 hold with $\varepsilon_{R,k} = 0$.

[15] This is because, by Proposition 3, the rate of growth of human capital is the same across the two models, which is the case only if the sum $\overline{u} + \overline{\nu}$ stays the same as in Uzawa-Lucas.

[16] In this case $\overline{u} + \overline{\nu}$ is larger and the growth rate of human capital is lower than in Uzawa-Lucas.

(accumulation of human capital), $1 - \overline{u} - \overline{\nu}$, is larger than in Uzawa-Lucas, while it is optimal to invest in financial literacy ($\overline{\nu} > 0$) only if $\varepsilon_{R,\nu} > 0$. The fastest growth of human capital increases economic growth by itself (the "human capital" channel), even if financial literacy does not influence the financial sector return.

Note that in this example we have abstracted from the effect of the aggregate level of financial literacy, a_t, as we have assumed that $\varepsilon_{R,a} = 0$. If the return generated by the financial sector positively depends on a_t ($\varepsilon_{R,a} > 0$) there is an additional incentive to invest in financial literacy, and this in the end amplifies growth through its effect on R.

Summarizing, acquiring new financial literacy per se does not amplify economic growth. It is not *necessary* to invest in financial literacy to increase growth via the "human capital channel". To invest in financial literacy is not *sufficient* to generate growth via the "financial return channel". What matters is the interaction between the investment in financial literacy and the financial sector's return. The "financial return channel" amplifies growth only if the financial sector return positively relates with the investment in new financial literacy, or with the existing level of financial knowledge.

As a priori we do not have information about the relative size of the elasticities of the financial sector return, both the "financial return" and the "human capital" channels are equally viable explanations of the importance of finance for economic growth.

5 Conclusion

This paper studies the impact of financial literacy on long-run equilibrium economic growth. We extend the Uzawa (1965) and Lucas (1988) endogenous growth model with the addition of a financial sector that transfers savings intertemporally. Furthermore, we allow agents to invest in financial literacy, defined as knowledge and skills specific to the financial sector. The financial sector efficiency depends on the macroeconomic conditions of the economy, and is therefore determined endogenously. Financial literacy is an intangible asset that requires human capital to be augmented. Both the acquisition of new financial literacy and its aggregate level do not affect the production of the final consumption good but might increase financial sector efficiency.

We show that embedding a financial sector in an otherwise standard Uzawa-Lucas model dramatically changes the main characteristics of this setup. The presence of a financial sector affects the long-term growth rate of the economy and the characteristics of its balanced growth path (BGP) equilibrium with respect to Uzawa-Lucas. Moreover, the financial sector generates a dynamic tradeoff concerning the allocation of existing human capital. On the cost side, it increases the opportunity cost of time for human capital accumulation. On the benefit side, it provides incentives to accumulate financial literacy if this has a positive effect on the financial sector return.

Based on this tradeoff, we identify two distinct ways through which finance can benefit economic growth. When the financial sector return is highly sensitive to the acquisition of new financial literacy, agents invest in it and reduce their human capital accumulation. If the financial sector is very effective in intertemporally transferring savings, the growth rate of the economy exceeds the one in a standard Uzawa-Lucas model. We call this the "financial return channel". Alternatively, the financial sector amplifies growth if its return strongly depends on the aggregate level of education (human capital). In this case it is optimal to let human capital grow faster than in the standard Uzawa-Lucas setup, and this in turn increases economic growth (the "human capital channel").

Finally, we clarify the role of financial literacy in spurring long run economic growth. Acquiring new financial literacy per se does not amplify economic growth. But financial literacy has a positive impact on long-term growth if the return earned by the financial sector positively relates with the investment in financial literacy, or with its aggregate level.

The relative size of the elasticities of the financial sector return with respect to its different inputs is crucial in assessing the effect of financial literacy and human capital on growth. We leave to further research

an accurate calibration of these key parameters.

A Technical Appendix

A.1 Proof of Proposition 1

The Bellman equation associated with problem (5) reads as follows:

$$V(k_t, h_t, a_t) = \max_{c_t, u_t, \nu_t} \{\ln c_t + \beta V(k_{t+1}, h_{t+1}, a_{t+1})\}$$

We look for an explicit expression of the value function by applying the "guess and verify" method. We therefore conjecture the following functional form:

$$V(k_t, h_t, a_t) = \theta + \theta_k \ln k_t + \theta_h \ln h_t + \theta_a \ln a_t$$

where θ_i with $i = \{k, h, a\}$ are parameters to be determined. From the above expression, after substituting the dynamic constraints, the Bellman equation reads as:

$$\begin{aligned} \theta + \theta_k \ln k + \theta_h \ln h + \theta_a \ln a &= \max_{c,u,\nu} \Big\{ \ln c + \beta\theta + \beta\theta_k \ln \big(R(.)(k^\alpha (uh)^{1-\alpha} - c)\big) + \beta\theta_h \ln \left(b(1-u-\nu)h\right) \\ &\quad + \beta\theta_a \ln \Big((\nu h)^{1-\xi} a^\xi \Big) \Big\} \end{aligned} \tag{24}$$

The FOCs follow:

$$\frac{1}{c} = \frac{\beta\theta_k}{k^\alpha (uh)^{1-\alpha} - c} \tag{25}$$

$$\beta\theta_k \left(\frac{\varepsilon_{R,u}}{u} + \frac{(1-\alpha)k^\alpha (uh)^{-\alpha} h}{k^\alpha (uh)^{1-\alpha} - c} \right) = \frac{\beta}{1-u-\nu}\theta_h \tag{26}$$

$$\frac{1}{\nu}(\theta_k \varepsilon_{R,\nu} + \theta_a (1-\xi)) = \theta_h \frac{1}{1-u-\nu} \tag{27}$$

where $\varepsilon_{R,u} = \frac{\frac{\partial R}{\partial u} u}{R}$, and $\varepsilon_{R,\nu} = \frac{\frac{\partial R}{\partial \nu} \nu}{R}$. The envelope conditions imply:

$$\frac{\theta_k}{k} = \left(\beta\varepsilon_{R,k} + \frac{\alpha\beta k^\alpha (uh)^{1-\alpha}}{k^\alpha (uh)^{1-\alpha} - c} \right) \frac{\theta_k}{k} \tag{28}$$

$$\frac{\theta_h}{h} = \beta\theta_k \left(\frac{(1-\alpha)k^\alpha (uh)^{-\alpha} u}{k^\alpha (uh)^{1-\alpha} - c} + \varepsilon_{R,h} \frac{1}{h} \right) + \beta\theta_h \frac{1}{h} + \beta\theta_a (1-\xi) \frac{1}{h} \tag{29}$$

$$\theta_a = \frac{\beta\varepsilon_{R,a}}{1-\beta\xi}\theta_k \tag{30}$$

where $\varepsilon_{R,k} = \frac{\frac{\partial R}{\partial k} k}{R}$. From (25) and (28) we obtain:

$$c = \frac{1-\alpha\beta-\beta\varepsilon_{R,k}}{1-\beta\varepsilon_{R,k}} y \tag{31}$$

$$\theta_k = \frac{\alpha}{1-\alpha\beta-\beta\varepsilon_{R,k}}, \tag{32}$$

while from (30) and (32):

$$\theta_a = \frac{\alpha\beta\varepsilon_{R,a}}{(1-\beta\xi)(1-\alpha\beta-\beta\varepsilon_{R,k})} \tag{33}$$

From (29), after plugging (25) and (32), we get:

$$\theta_h = \frac{(1-\alpha)\big(1-\alpha\beta-\beta\varepsilon_{R,k}\big) + \alpha\beta\Big(1-\alpha+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\Big)}{(1-\beta)\big(1-\alpha\beta-\beta\varepsilon_{R,k}\big)} \tag{34}$$

The three expressions (32), (33), and (34) determine the parameters in the value function V, while the constant θ follows from solving (24) after substituting all relevant expressions.

From (29) and (26), and by using (27), we have:

$$u+\nu=\overline{u}+\overline{\nu}=1-\beta\frac{\theta_h}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{35}$$

By combining (27) and (26) with (35), after tedious algebra it is possible to obtain:

$$\nu = \overline{\nu}=\frac{\beta\left(\theta_k\varepsilon_{R,\nu}+(1-\xi)\,\theta_a\right)}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{36}$$

$$u = \overline{u}=\frac{\beta\theta_k\varepsilon_{R,u}+(1-\alpha)\,(1+\beta\theta_k)}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{37}$$

From (34) and (32) we have:

$$1-\overline{u}-\overline{\nu}=\beta\left(1-\frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \tag{38}$$

while from (36) and (37):

$$\frac{\overline{\nu}}{\overline{u}}=\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{39}$$

By combining the previous two expression it is possible to determine:

$$\overline{u}=\frac{1-\beta\left(\frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}} \tag{40}$$

which can be written as (7) by defining

$$\Theta = \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{41}$$

$$\Delta = \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{42}$$

We substitute (40) into (39) to obtain

$$\overline{\nu}=\frac{1-\beta\left(\frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}}$$

which can be written as (8) by the use of Θ and Δ defined in (41) and (42) respectively.

We now turn to the conditions on the model parameters and on the elasticities such that our solution has economic relevance, i.e. $c_t \geq 0$, $\overline{u}\in(0,1)$ and $\overline{\nu}\in[0,1)$, and $\overline{u}+\overline{\nu}\in(0,1)$.

Suppose that all elasticities $\varepsilon_{R,i}\geq 0, i=\{k,h,a,u,\nu\}$.

Then from (6) it is immediate to see that $c_t\geq 0$ if $\varepsilon_{R,k}\leq\frac{1-\alpha\beta}{\beta}$.

We now move to the conditions under which $\overline{u}>0$.

From (7), we have that $\overline{u}>0$ if both $\beta\Theta<1$ and $\Delta>0$, with Θ and Δ in (41) and (42). A sufficient condition for $\Delta>0$ is $\varepsilon_{R,k}\leq\frac{1}{\beta}$, given that all other elasticites $\varepsilon_{R,u}$, $\varepsilon_{R,\nu}$ and $\varepsilon_{R,a}$ are non-negative, and

$\alpha < 1$, $\xi < 1$.

In order to verify that $\Theta < \frac{1}{\beta}$, first notice that a sufficient condition for Θ's numerator to be positive is $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta} < \frac{1-\alpha\beta}{\beta} < \frac{1}{\beta}$.[17] We proceed by distinguishing two cases:

(i) $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$.

The condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ is sufficient to guarantee that also Θ's denominator is positive so that

$$\Theta = \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \leq 1 < \frac{1}{\beta}$$

and $\overline{u} > 0$.

(ii) $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$.

In order to verify that $\Theta < \frac{1}{\beta}$ we first need to determine the sign of Θ's denominator, which is positive if

$$\begin{aligned}
1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) &> -\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\\
1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)-\beta\varepsilon_{R,k}(1-\alpha) &> -\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\\
1-\alpha+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta^2\left(\varepsilon_{R,h}+\frac{1-\xi}{1-\beta\xi}\varepsilon_{R,a}\right) &> \beta\varepsilon_{R,k}(1-\alpha)
\end{aligned}$$

The LHS of the above inequality is larger than $(1-\alpha)$ because all $\varepsilon_{R,u}$, $\varepsilon_{R,\nu}$, $\varepsilon_{R,h}$, and $\varepsilon_{R,a}$ are non-negative, while the RHS is lower than $(1-\alpha)$ because $\varepsilon_{R,k} < \frac{1}{\beta}$. Then Θ's denominator is positive. Therefore, the condition $\Theta < \frac{1}{\beta}$ is equivalent to

$$\begin{aligned}
1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\tfrac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) &< \tfrac{1}{\beta}\left(\begin{array}{c}1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)\\ +\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\end{array}\right)\\
(1-\alpha)(1-\beta)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\tfrac{\alpha\beta^2(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} &> -\beta^2(1-\alpha)\varepsilon_{R,k}
\end{aligned}$$

which always holds, given that $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq 0$, $\varepsilon_{R,a} \geq 0$ and $\varepsilon_{R,k} \geq 0$.

Summarizing, $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ is sufficient to guarantee that both $\beta\Theta < 1$ and $\Delta > 0$ and therefore $\overline{u} > 0$.

Now we analyze the conditions under which $\overline{\nu} \geq 0$.

From (39) we write $\overline{\nu} = \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}\overline{u}$. Given that $\varepsilon_{R,\nu} \geq 0$, $\varepsilon_{R,a} \geq 0$, and $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ (so that $\overline{u} > 0$ by the discussion above), one can see that $\overline{\nu} \geq 0$ if $\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}(1-\beta\varepsilon_{R,k}) > 0$, which is verified with $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$.

Finally, we study the conditions under which both $\overline{u} < 1$ and $\overline{\nu} < 1$. Having established that both $\overline{u} > 0$ and that $\overline{\nu} \geq 0$, a sufficient condition for $\overline{u} < 1$ and $\overline{\nu} < 1$ is $\overline{u} + \overline{\nu} < 1$.

By (38),

$$\begin{aligned}
\overline{u}+\overline{\nu} &= 1-\beta\left(1-\frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)\\
&= 1-\beta\left(\frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)\\
&= 1-\beta\Theta \qquad (43)
\end{aligned}$$

from (41). The condition $\overline{u}+\overline{\nu} < 1$ reduces to $\Theta > 0$. We have shown above that both Θ's numerator and Θ's

[17] Therefore, the condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ also guarantees that $c_t \geq 0$ and that $\Delta > 0$.

denominator are positive when $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$, irrespectively whether $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ or $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$. This concludes the proof that $\overline{u} + \overline{\nu} < 1$.

We now derive the comparative statics results, starting from $\overline{u}$, which we write as in (7), so that, for every $\varepsilon_{R,i}$, $i = \{h, a, u, \nu\}$:

$$\frac{\partial \overline{u}}{\partial \varepsilon_{R,i}} = \frac{-\beta \frac{\partial \Theta}{\partial \varepsilon_{R,i}} \Delta - (1 - \beta\Theta) \frac{\partial \Delta}{\partial \varepsilon_{R,i}}}{\Delta^2} \tag{44}$$

Let us derive:

$$\begin{aligned}
\frac{\partial \Theta}{\partial \varepsilon_{R,h}} &= \frac{\left(\begin{array}{c} \alpha\beta \left(1 - \alpha - \beta\varepsilon_{R,k} + \alpha\beta \left(\varepsilon_{R,k} + \varepsilon_{R,h} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) + \alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h}\right)\right) - \\ \left(1 - \alpha - \beta\varepsilon_{R,k} + \alpha\beta \left(\varepsilon_{R,k} + \varepsilon_{R,h} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)\right)(\alpha\beta - \alpha\beta(1-\beta)) \end{array} \right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} \\
&= \frac{\alpha^2\beta^2(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)+\alpha\beta(1-\beta)\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)\right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} \\
&= \frac{\alpha\beta(1-\beta)\left(\alpha\beta\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}+1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\varepsilon_{R,k}\right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} > 0
\end{aligned}$$

given that $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ and all other elasticities are non-negative, while $\frac{\partial \Delta}{\partial \varepsilon_{R,h}} = 0$. From (44), we obtain that

$$\frac{\partial \overline{u}}{\partial \varepsilon_{R,h}} < 0$$

Next,

$$\begin{aligned}
\frac{\partial \Theta}{\partial \varepsilon_{R,u}} &< 0 \\
\frac{\partial \Delta}{\partial \varepsilon_{R,u}} &= \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)-\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)^2} \\
&= -\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)^2} < 0
\end{aligned}$$

and from (44) we can conclude that

$$\frac{\partial \overline{u}}{\partial \varepsilon_{R,u}} > 0$$

Concerning the comparative statics of $\overline{\nu}$, we start by considering again (8)

$$\begin{aligned}
\overline{\nu} &= \frac{1-\beta\Theta}{\Delta} \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \\
&= \frac{1-\beta\Theta}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}} \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \\
&= \frac{1-\beta\Theta}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}} = \frac{1-\beta\Theta}{\Delta_1}
\end{aligned}$$

and, as shown before, $\frac{\partial\Theta}{\partial\varepsilon_{R,h}} > 0$, $\frac{\partial\Theta}{\partial\varepsilon_{R,u}} = \frac{\partial\Theta}{\partial\varepsilon_{R,\nu}} < 0$. From $\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,i}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,i}}\Delta_1-(1-\beta\Theta)\frac{\partial\Delta_1}{\partial\varepsilon_{R,i}}}{\Delta_1^2}$ we obtain

$$\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,h}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,h}}\Delta_1}{\Delta_1^2} < 0$$

because Δ_1 is independent of $\varepsilon_{R,h}$, and

$$\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,\nu}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,\nu}}\Delta_1 - (1-\beta\Theta)\frac{\partial\Delta_1}{\partial\varepsilon_{R,\nu}}}{\Delta_1^2} > 0$$

because

$$\frac{\partial\Delta_1}{\partial\varepsilon_{R,\nu}} = \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}-\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2} = -\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2} < 0$$

We also derive the comparative statics w.r. to $\varepsilon_{R,a}$. Starting from $\frac{\partial\overline{u}}{\partial\varepsilon_{R,a}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,a}}\Delta-(1-\beta\Theta)\frac{\partial\Delta}{\partial\varepsilon_{R,a}}}{\Delta^2}$ and

$$\begin{aligned}
\frac{\partial\Theta}{\partial\varepsilon_{R,a}} &= \frac{(\alpha\beta)^2\frac{\beta(1-\xi)(1-\beta)}{1-\beta\xi}\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} \\
\frac{\partial\Delta}{\partial\varepsilon_{R,a}} &= \frac{\frac{\beta(1-\xi)}{1-\beta\xi}}{\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)^2} > 0
\end{aligned}$$

one has

$$\frac{\partial\overline{u}}{\partial\varepsilon_{R,a}} = \begin{cases} < 0 \text{ if } \frac{\partial\Theta}{\partial\varepsilon_{R,a}} > 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h} \\ < 0 \text{ if } \frac{\partial\Theta}{\partial\varepsilon_{R,a}} = 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} = \varepsilon_{R,h} \\ ??? \ \frac{\partial\Theta}{\partial\varepsilon_{R,a}} < 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} > \varepsilon_{R,h} \end{cases}$$

and for $\overline{\nu} = \frac{1-\beta\Theta}{\Delta_1}$:

$$\frac{\partial\Delta_1}{\partial\varepsilon_{R,a}} = \frac{-\frac{\beta(1-\xi)}{1-\beta\xi}\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2} < 0$$

if $\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right) \geq 0$ (which is guaranteed by $\varepsilon_{R,k} > \frac{1}{\beta}$). Therefore, $\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,a}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,a}}\Delta_1-(1-\beta\Theta)\frac{\partial\Delta_1}{\partial\varepsilon_{R,a}}}{\Delta^2}$

$$\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,a}} = \begin{cases} > 0 \text{ if } \frac{\partial\Theta}{\partial\varepsilon_{R,a}} < 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} > \varepsilon_{R,h} \\ > 0 \text{ if } \frac{\partial\Theta}{\partial\varepsilon_{R,a}} = 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} = \varepsilon_{R,h} \\ ??? \ \frac{\partial\Theta}{\partial\varepsilon_{R,a}} > 0, \text{ i.e. if } \varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h} \end{cases}$$

■

A.2 Proof of Proposition 2

The fact that both the share of human capital allocated to production, u_t and to financial literacy accumulation, ν_t, are constant simply derives from (7)-(8) in Proposition 1, with $\varepsilon_{R,k} = 0$:

$$\begin{aligned}
\overline{u} &= \frac{1-\beta\Theta'}{\Delta'} \\
\overline{\nu} &= \frac{1-\beta\Theta'}{\Delta'}\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}}
\end{aligned}$$

where

$$\Theta' = \frac{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{45}$$

$$\Delta' = \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}} \tag{46}$$

Notice that $\varepsilon_{R,k} = 0 \leq \frac{1-\alpha}{\beta}$, given that $\alpha < 1$. Therefore, by Proposition 1, we have that $\overline{u} \in (0,1)$, $\overline{\nu} \in [0,1)$ and $\overline{u}+\overline{\nu} < 1$.

Again by Proposition 1 (see (12)) one obtains $\frac{h_{t+1}}{h_t} = 1+\gamma_h = b(1-\overline{u}-\overline{\nu})$. Therefore the long-run rate of growth of human capital in our model γ_h is positive only if $b > \frac{1}{1-\overline{u}-\overline{\nu}}$. The expression for $\overline{u}+\overline{\nu}$ can be obtained from (43) with $\varepsilon_{R,k} = 0$:

$$\begin{aligned}
\overline{u}+\overline{\nu} &= 1-\beta\left(1-\frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \\
&= 1-\beta\left(\frac{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \\
&= 1-\beta\Theta'
\end{aligned}$$

hence

$$1-(\overline{u}+\overline{\nu}) = \beta\Theta' \tag{47}$$

and therefore

$$\gamma_h = b\beta\Theta' - 1$$

We have $\gamma_h > 0$ if $b > \frac{1}{\beta\Theta'}$.

It is easy to verify that a_t grows at the same rate as human capital, $\gamma_a = \gamma_h$:

$$1+\gamma_a = \frac{a_{t+1}}{a_t} = \frac{\overline{\nu}^{1-\xi}h_t^{1-\xi}a_t^{\xi}}{\overline{\nu}^{1-\xi}h_{t-1}^{1-\xi}a_{t-1}^{\xi}} = \left(\frac{h_t}{h_{t-1}}\right)^{1-\xi}\left(\frac{a_t}{a_{t-1}}\right)^{\xi} = (1+\gamma_h)^{1-\xi}(1+\gamma_a)^{\xi}$$

which holds only if $\gamma_a = \gamma_h$, when both γ_h and γ_a are strictly positive.

Let γ_k be the rate of growth of physical capital k_t. By using $u_t = \overline{u}$ and the budget constraint for physical capital in (5), one obtains

$$1+\gamma_k = \frac{k_{t+1}}{k_t} = \frac{R_t\alpha\beta\overline{u}^{1-\alpha}k_t^{\alpha}h_t^{1-\alpha}}{R_{t-1}\alpha\beta\overline{u}^{1-\alpha}k_{t-1}^{\alpha}h_{t-1}^{1-\alpha}}$$

where R_t is the return on savings generated at time t, i.e. $R(h_t, a_t, \overline{u}, \overline{\nu})$, while R_{t-1} is the return on savings generated at $t-1$, i.e. $R(h_{t-1}, a_{t-1}, \overline{u}, \overline{\nu})$. Therefore:

$$\begin{aligned}
1+\gamma_k &= \frac{R_t}{R_{t-1}}\left(\frac{k_t}{k_{t-1}}\right)^{\alpha}\left(\frac{h_t}{h_{t-1}}\right)^{1-\alpha} \\
1+\gamma_k &= \frac{R_t}{R_{t-1}}(1+\gamma_k)^{\alpha}(1+\gamma_h)^{1-\alpha}
\end{aligned}$$

and solving for $1+\gamma_k$:

$$1+\gamma_k = \left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{1-\alpha}}(1+\gamma_h) = (1+\gamma_R)^{\frac{1}{1-\alpha}}(1+\gamma_h) \tag{48}$$

where γ_R is the growth rate of the financial sector' return.

Given that the return function $R(h, a, \overline{u}, \overline{\nu})$ is strictly increasing in all its arguments, $\gamma_R \geq 0$ as long as h_t and a_t are non-decreasing. We have obtained above that $\gamma_a = \gamma_h > 0$ if $b > \frac{1}{\beta\Theta'}$, and therefore under this condition $\gamma_R \geq 0$. From (48) one derives that $\gamma_k > 0$.

As for the growth rate of production γ_y (and consumption, since with $\varepsilon_{R,k} = 0$ the optimal consumption $c_t = (1 - \alpha\beta)y_t$):

$$\begin{aligned}
1 + \gamma_y &= \frac{y_t}{y_{t-1}} = \frac{\overline{u}^{1-\alpha} k_t^{\alpha} h_t^{1-\alpha}}{\overline{u}^{1-\alpha} k_{t-1}^{\alpha} h_{t-1}^{1-\alpha}} = \left(\frac{k_t}{k_{t-1}}\right)^{\alpha} \left(\frac{h_t}{h_{t-1}}\right)^{1-\alpha} \\
&= (1 + \gamma_k)^{\alpha} (1 + \gamma_h)^{1-\alpha} \\
&= \left(\frac{R_t}{R_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} (1 + \gamma_h)^{\alpha} (1 + \gamma_h)^{1-\alpha} \\
&= \left(\frac{R_t}{R_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} (1 + \gamma_h) = (1 + \gamma_R)^{\frac{\alpha}{1-\alpha}} (1 + \gamma_h) \qquad (49)
\end{aligned}$$

by using (48).

Concerning the comparative statics of γ_h w.r. to the elasticities of R, from (18) we have:

$$\frac{\partial \gamma_h}{\partial \varepsilon_{R,i}} = b\beta \frac{\partial \Theta'}{\partial \varepsilon_{R,i}}$$

for all $\varepsilon_{R,i} = \{\varepsilon_{R,h}, \varepsilon_{R,a}, \varepsilon_{R,u}, \varepsilon_{R,\nu}\}$. Moreover,

$$\begin{aligned}
\frac{\partial \Theta'}{\partial \varepsilon_{R,h}} &= \frac{\alpha\beta\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)+\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)\right)(\alpha\beta-\alpha\beta(1-\beta))}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} \\
&= \frac{\alpha\beta^2\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta\left(1-\alpha+\alpha\beta\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} > 0
\end{aligned}$$

while it is immediate to see from (45) that $\frac{\partial \Theta'}{\partial \varepsilon_{R,u}} = \frac{\partial \Theta'}{\partial \varepsilon_{R,\nu}} < 0$.

For completeness, we also compute

$$\begin{aligned}
\frac{\partial \Theta'}{\partial \varepsilon_{R,a}} &= \frac{\frac{\alpha\beta^2(1-\xi)}{1-\beta\xi}\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)-\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)\right)\frac{\alpha\beta^2(1-\xi)}{1-\beta\xi}}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} \\
&= \frac{\alpha\beta(1-\beta)\frac{\alpha\beta^2(1-\xi)}{1-\beta\xi}}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2}\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)
\end{aligned}$$

whose sign corresponds to the sign of $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h}$. Therefore:

$$\begin{aligned}
\frac{\partial \gamma_h}{\partial \varepsilon_{R,a}} &\geq 0 \text{ if } \varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h} \\
\frac{\partial \gamma_h}{\partial \varepsilon_{R,a}} &< 0 \text{ if } \varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}
\end{aligned}$$

■

A.3 Proof of Proposition 3

We start by recalling the BGP in a standard Uzawa-Lucas model without financial sector. The long-run rate of growth of human capital $\frac{h_{t+1}}{h_t} = 1 + \gamma_h^{UL} = 1 + \gamma^{UL} = b(1 - \overline{u}) = b\beta$, given that $1 - \overline{u} = \beta$, with

$\gamma^{UL} > 0$ if $b > \frac{1}{\beta}$.

Let us fix the same b across the two models. From (12) and (47), we have that in our framework $1 + \gamma_h = b\beta\Theta'$, and $\gamma_h > 0$ if $b > \frac{1}{\beta\Theta'}$. It is straightforward to see that $\gamma_h > \gamma^{UL}$ ($\gamma_h \leq \gamma^{UL}$) if $\Theta' > 1$ ($\Theta' \leq 1$). For simplicity, let us define $\Gamma = 1 - \Theta'$, that is:

$$\Gamma = 1 - \frac{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} = \frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}$$

We have that $\gamma_h > \gamma^{UL}$ ($\gamma_h \leq \gamma^{UL}$) if $\Gamma < 0$ ($\Gamma \geq 0$).

Let us study the sign of Γ. We start from its denominator. Given that all elasiticites are non-negative and $\alpha < 1$, Γ's denominator is always positive if $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} \geq 0$. If $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} < 0$ we have that

$$\begin{aligned} 1 - \alpha + \alpha\beta\left(\varepsilon_{R,h} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) + \alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h}\right) &> 0 \\ 1 - \alpha + \alpha\beta\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} + \alpha\beta^2\varepsilon_{R,h} &> -\alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu}\right) \end{aligned}$$

A sufficient condition for the above inequality to be satisfied is

$$\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} + \beta\varepsilon_{R,h} > 0$$

because $1 - \alpha > 0$ and both $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$ are non-negative. This always holds because also $\varepsilon_{R,a}$ and $\varepsilon_{R,h}$ are non-negative. Therefore, we can conclude that $den(\Gamma) > 0$ and $sgn(\Gamma) = sgn(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h})$. Summarizing:

- if $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\Gamma \geq 0$ and $\gamma_h \leq \gamma^{UL}$;
- if $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ then $\Gamma < 0$ and $\gamma_h > \gamma^{UL}$.

Let us move to the comparison of γ_y with γ^{UL}. Given (20), it is straightforward to verify that $\gamma_y > \gamma^{UL}$ as long as $\gamma_R \geq 0$ and $\gamma_h > \gamma^{UL}$, that is when $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$.

If $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\gamma_h \leq \gamma^{UL}$ and we have that $\gamma_y \geq \gamma^{UL}$ only if

$$1 + \gamma_y = (1 + \gamma_R)^{\frac{\alpha}{1-\alpha}}(1 + \gamma_h) \geq 1 + \gamma^{UL}$$

We substitute for $1 + \gamma_h = b\beta\Theta'$ obtained above and for $\gamma^{UL} = b\beta - 1$:

$$\begin{aligned} (1 + \gamma_R)^{\frac{\alpha}{1-\alpha}} b\beta\Theta' &\geq b\beta \\ 1 + \gamma_R &\geq \left(\frac{1}{\Theta'}\right)^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

that proves the proposition.[18]

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[18] Recall that $\Theta' < 1$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$. In this case, $\gamma_h < \gamma^{UL}$ and in order to obtain that $\gamma_y > \gamma^{UL}$ we need that γ_R is "sufficiently high" ($1/\Theta' > 1$).

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