GFLEC
GLOBAL FINANCIAL LITERACY
EXCELLENCE CENTER

The Time Value of Money

Lecture 2

The interest compounding formula

In general, the interest compounding formula is:

$$F = P(1+r)^T$$

Where:

P = starting amount (principal)

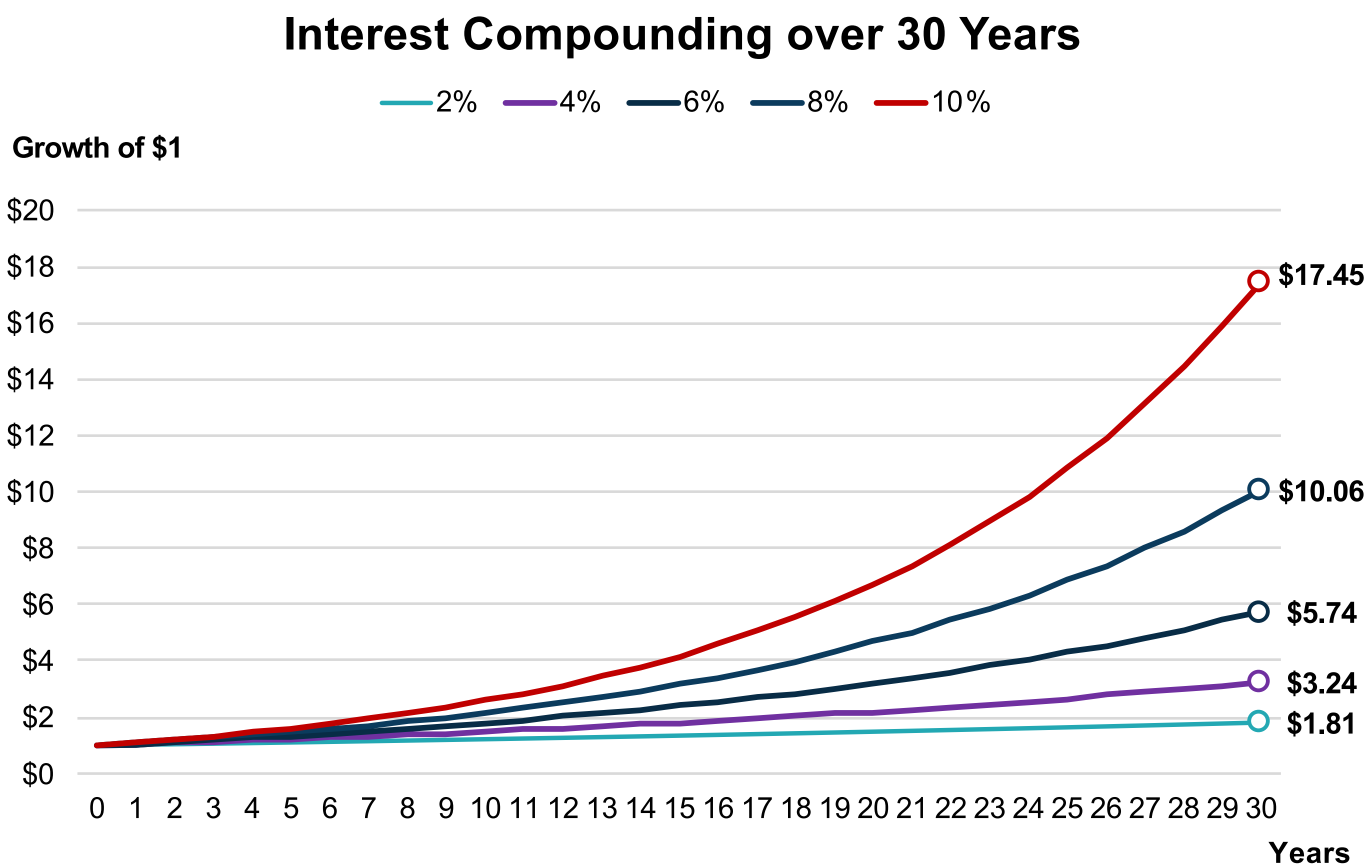
r = interest rate

T = time periods

F = final amount

Investing at a high interest rate

The higher the interest rate, the larger the effect of interest compounding over time.



Time is very important

Time is money:

- Opportunity cost
- We can look at how money grows at time T
- We can look at money at time 0

$100K today or in a year?

Would you rather receive $100,000 today or $100,000 one year from now?

Today

$100,000

OR

One Year

$100,000

a) $100,000 today
b) $100,000 in one year
c) It's the same amount, so it doesn't matter
d) It depends

$100K today or in a year?

You should prefer $100,000 today.

Today

$100,000

One Year

$100,000

a) **$100,000 today**
b) $100,000 in one year
c) It's the same amount, so it doesn't matter
d) It depends

$100K today or in a year?

This is because a dollar today can be invested and earns interest.

<u>Today</u>

Dollar

Invest

<u>One Year</u>

Dollar + interest

- It's better to have a dollar today than in the future. You can invest it and get more than a dollar in the future.
- **A dollar today is worth more than a dollar in the future**.

The time value of money

A dollar today is more valuable than a dollar later.

This is called the **time value of money**…

$100K today or $110K in a year?

Let's do a trickier one: would you rather receive $100,000 today or *$110,000* one year from now?

Today

$100,000

OR

One Year

$110,000

a) $100,000 today, because I need money today
b) $110,000 in one year because it's more
c) It depends

$100K today or $110K in a year?

"It depends"

It depends on the interest rate!

If the interest rate is **5%**, it would be better to **take $110,000 in one year**, because $100,000 will not grow to $110,000, even after interest.

$$\$100{,}000 * (1 + r) = \$100{,}000 * 1.05 = \$105{,}000$$

- But if the interest rate is **20%**, it would be better to **take the $100,000 today**, because it will grow to more than $110,000 after one year.

$$\$100{,}000 * (1 + r) = 100{,}000 * 1.20 = \$120{,}000$$

$100K today or $110K in a year?

"What if I need the money today?"

Then borrow!

- If the interest rate is **5%**, still **take $110,000 in one year**, but borrow $100,000 today.
- After one year, you will owe $105,000.
- You can use the $110,000 in one year to repay the $105,000 and still have $5,000 left over.
- You get your $100,000 today and also $5,000 in one year!
- But if the interest rate is **20%**, **take the $100,000 today**. If you borrow, you will owe more than $110,000 in one year.

Present Value

Moving money through time

Consumers can “move” money across time by saving and borrowing.

- Consumers often move their money into the future, for example by saving for retirement.
- Consumers also use their future income today by taking out loans, for example to purchase a home today.
- But as we saw earlier, the value of money changes at different points in time

- So we need a way to compare the value of cash flows received at different times
- To do so, we will introduce the concept of **present value**

Present value

The **present value** of a future cash flow is the value of that cash flow in today's dollars.

Earlier, we found that $110,000 in one year is worth $100,000 today when the interest rate is 10%. So, in finance, we say:

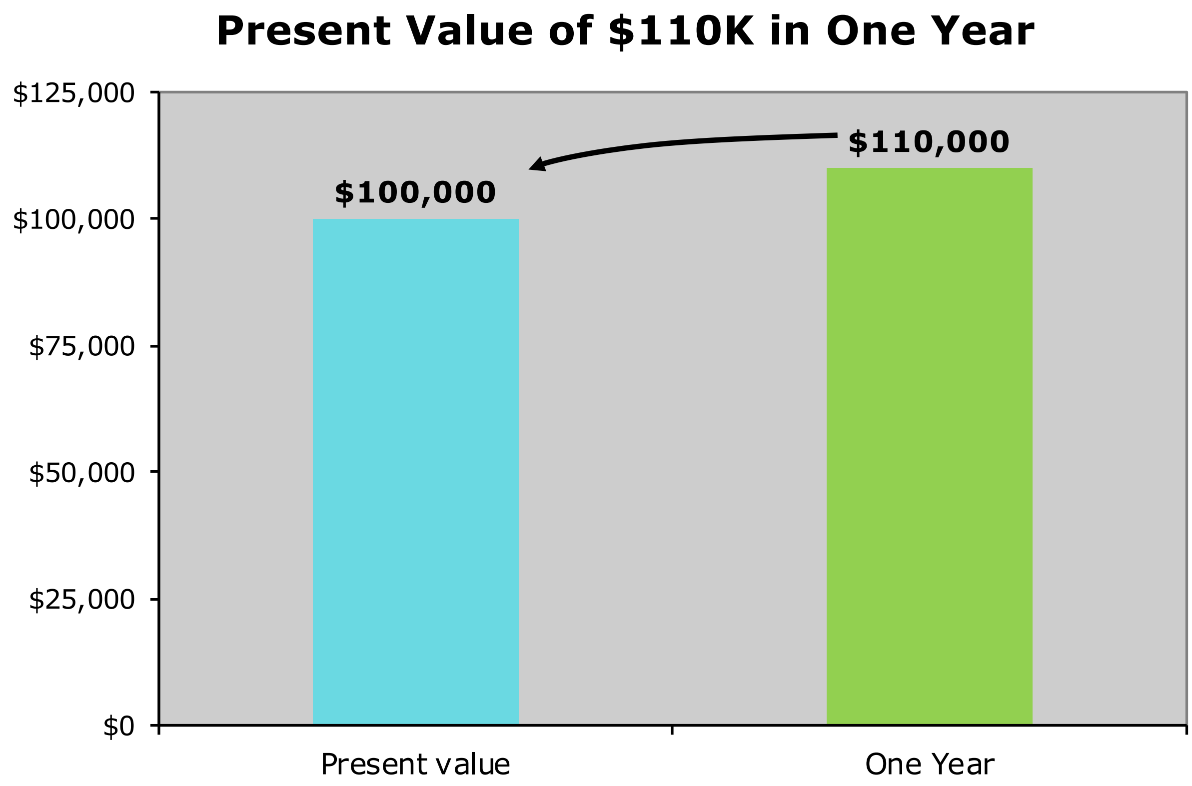
> The **present value** of $110,000 in one year is $100,000.
> (when the interest rate is 10%)

Notice that $110,000 to be received in the future is worth less than its face value today.

This is generally true, and the further in the future the $110,000 will be received, the less it will be worth today.

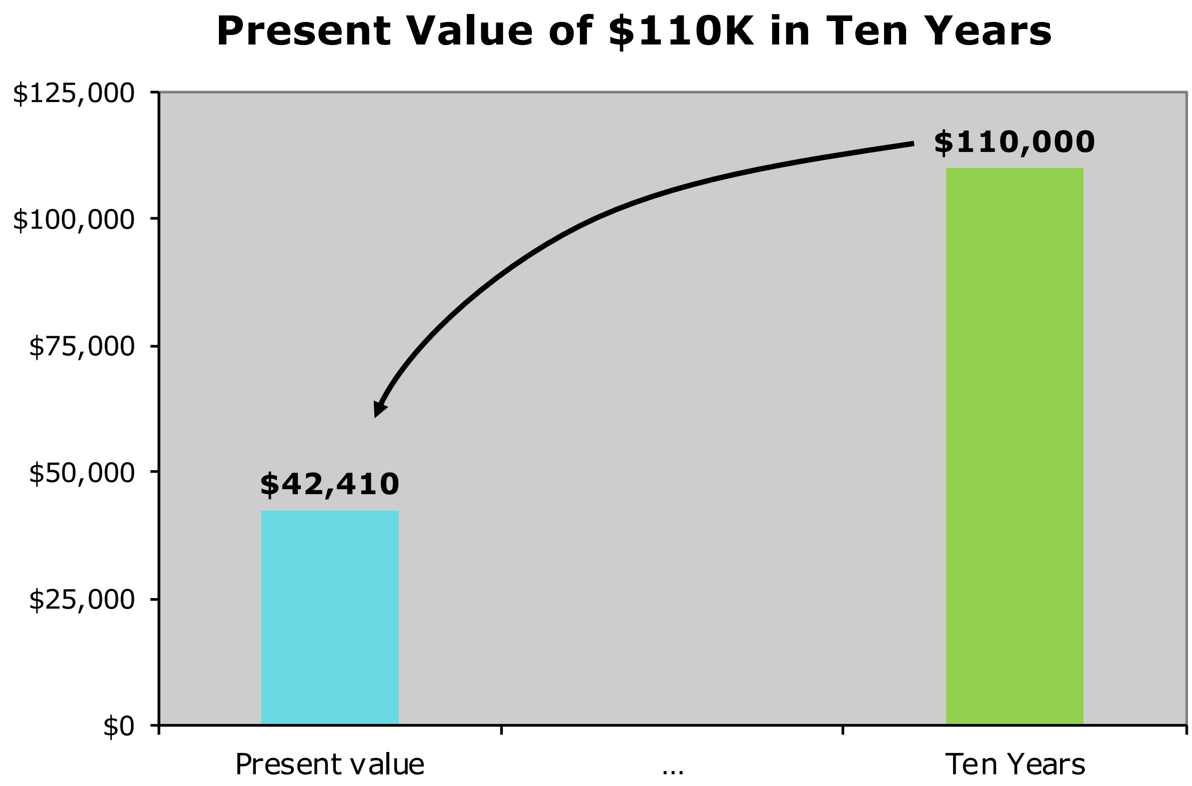
Visualizing the time value of money

$110K to be received in one year is worth $100K today if the interest rate is 10%.



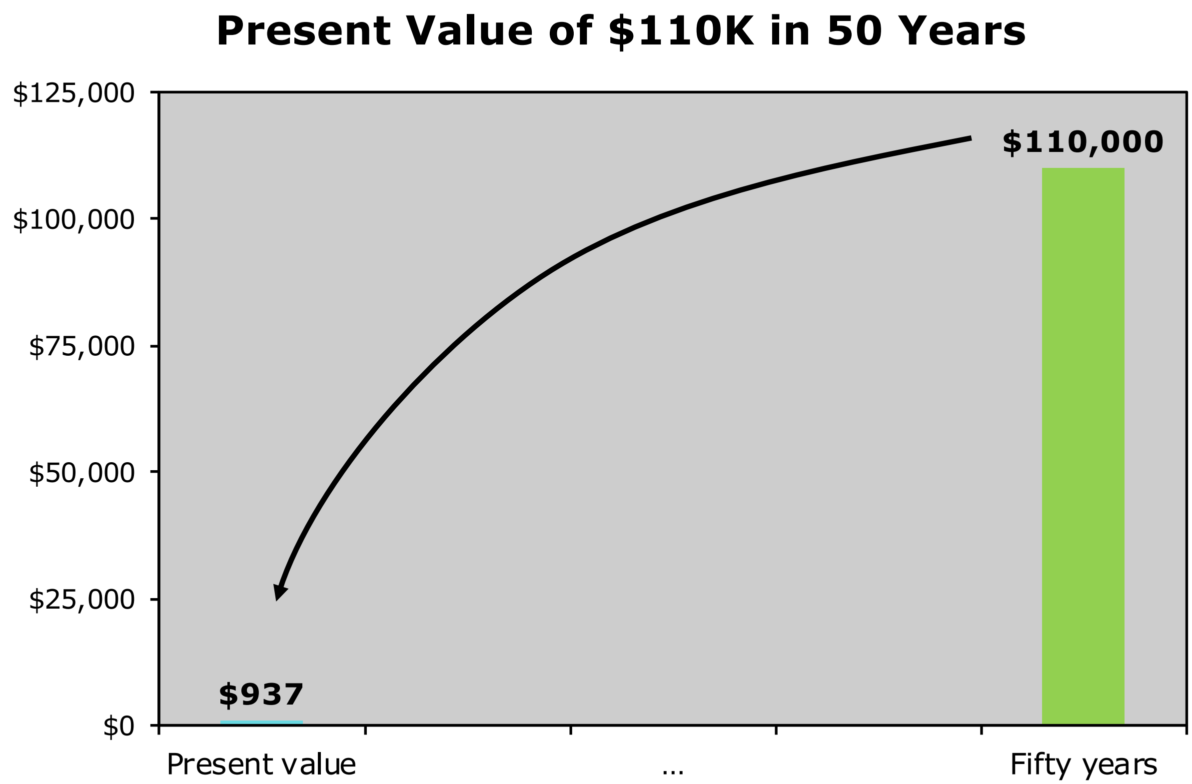
Visualizing the time value of money

But worth only $42,000 if it will be received in *ten years*…



Visualizing the time value of money

And worth only $900 if it will be received in 50 years!



The present value formula

In general, the present value *PV* can be found by rearranging the interest compounding formula:

$$F = PV(1+r)^T$$

$$\downarrow$$

$$\boxed{PV = \frac{F}{(1+r)^T}}$$

Where:

PV = present value

F = future cash flow received at time T

r = interest rate

Discounting and the time value of money

Computing the present value is called **discounting**, and it is the opposite of **interest compounding**.

From the present value formula we see that the **time value of money** is the power of interest compounding in reverse!

$$PV = \frac{F}{(1+r)^T}$$

As a dollar is pushed further into the future, *T* becomes larger, and its present value becomes smaller.

And the higher the interest rate *r*, the more quickly it shrinks.

Valuing a lottery payout

Many state lotteries require the winner to receive payments spread into the future to collect the full jackpot. If the winner wants the cash today, he or she may only receive a fraction of the winnings.

Ex. Michael wins the state lottery with a $250,000 jackpot. To receive the full jackpot, he must wait five years. If he collects his winnings today, he will only receive $200,000. If the interest rate is 6%, what is the better deal?

Ans. The present value of the $250,000 jackpot in five years can be found by discounting the sum by 6%:

$$PV = \frac{F}{(1+r)^T} = \frac{\$250{,}000}{1.06^5} = \$186{,}815$$

Because this is less than $200,000, it is better to take the $200,000 today.

Present value of a stream of cash flows

In many situations, there are multiple cash flows to value.

The present value of a stream of cash flows is the sum of the present values of the individual cash flows:

$$PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

Where C_T is the cash flow at time T.

Applications of present value

Situations where you might have to find the present value of a stream of cash flows include:

- **Establishing a business:** A good business will generate income each year, but will usually require an up-front investment. To determine whether the business is worth starting, the present value of the income it generates should be compared to the cost of the investment today.

- **Pursuing education**: A good education will increase a graduate's earnings over a lifetime. To determine if a degree is financially valuable, compare the tuition cost to the present value of the increase in future earnings.

- **Valuing financial assets:** A financial asset, such as a stock or bond, is a contract that promises payments to the holder. To determine if an asset is worth buying, compare its price to the present value of its promised payments.

Pricing a Bond

What is a bond?

A bond is an example of financial asset.

- A bond may be issued by a **corporation or government**, and effectively acts as a loan to the issuer.

- For long-term bonds, the issuer issues the security to investors in exchange for cash. The issuer then pays periodic interest on the bond in installments known as **coupon payments** and, at the bond's maturity, the issuer also pays back the original **face value** of the bond.

THE UNITED STATES OF AMERICA

FOR VALUE RECEIVED PROMISES TO PAY TO

1

-THE FIRST NATIONAL BANK OF BOSTON,
BOSTON, MASSACHUSETTS-

2½%

TREASURY BOND
of 1963

Dated December 15, 1954
Due August 15, 1963

OR REGISTERED ASSIGNS THE SUM OF

ONE HUNDRED THOUSAND DOLLARS

ON AUGUST 15, 1963, AND TO PAY INTEREST ON SAID PRINCIPAL SUM FROM DEC 15 1954 , AT THE RATE OF TWO AND ONE-HALF PERCENT PER ANNUM, PAYABLE ON A SEMIANNUAL BASIS ON AUGUST 15, 1955, AND THEREAFTER ON FEBRUARY 15 AND AUGUST 15 IN EACH YEAR UNTIL THE PRINCIPAL HEREOF SHALL BE PAYABLE, AT THE TREASURY DEPARTMENT, WASHINGTON, D. C., OR, AT THE HOLDER'S OPTION, AT ANY AGENCY OR AGENCIES IN THE UNITED STATES WHICH THE SECRETARY OF THE TREASURY MAY FROM TIME TO TIME DESIGNATE FOR THE PURPOSE. THIS BOND IS ONE OF A SERIES OF BONDS OF THE UNITED STATES, AUTHORIZED BY THE SECOND LIBERTY BOND ACT, AS AMENDED, ISSUED PURSUANT TO AND SUBJECT TO THE TERMS AND CONDITIONS OF TREASURY DEPARTMENT CIRCULAR No. 952, DATED NOVEMBER 22, 1954, AND DESIGNATED 2½ PERCENT TREASURY BONDS OF 1963. THIS BOND IS NOT SUBJECT TO CALL FOR REDEMPTION PRIOR TO MATURITY. THE INCOME DERIVED FROM THIS BOND SHALL BE SUBJECT TO ALL TAXES NOW OR HEREAFTER IMPOSED UNDER THE INTERNAL REVENUE CODE, OR LAWS AMENDATORY OR SUPPLEMENTARY THERETO. THIS BOND SHALL BE SUBJECT TO ESTATE, INHERITANCE, GIFT OR OTHER EXCISE TAXES, WHETHER FEDERAL OR STATE, BUT SHALL BE EXEMPT FROM ALL TAXATION NOW OR HERE-AFTER IMPOSED ON THE PRINCIPAL OR INTEREST HEREOF BY ANY STATE, OR ANY OF THE POSSESSIONS OF THE UNITED STATES, OR BY ANY LOCAL TAXING AUTHORITY. THIS BOND IS ACCEPTABLE TO SECURE DEPOSITS OF PUBLIC MONEYS. THIS BOND IS SUBJECT TO THE GENERAL REGULATIONS OF THE TREASURY DEPARTMENT WITH RESPECT TO UNITED STATES BONDS.

WASHINGTON, D. C., DECEMBER 15, 1954.

SECRETARY OF THE TREASURY

TRANSFERABLE ON THE BOOKS OF THE UNITED STATES TREASURY DEPARTMENT.

ONE HUNDRED THOUSAND DOLLARS

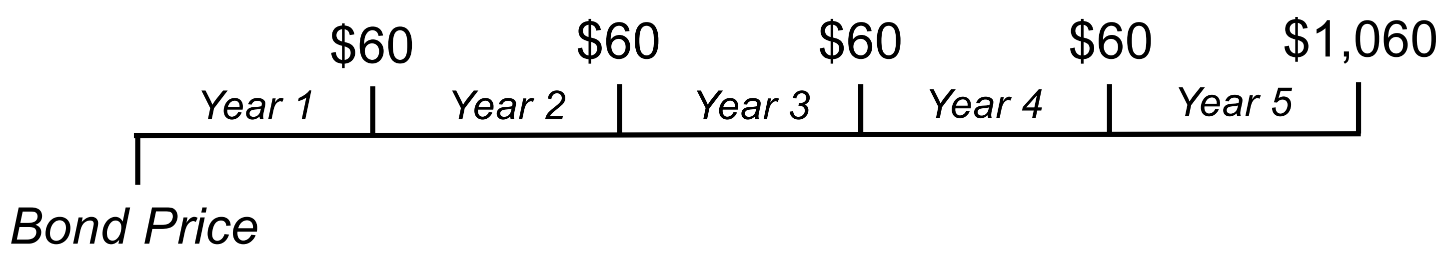
Bureau of Engraving and Printing

What is a bond?

Ex. A corporation issues a bond with a **face value of $100 million that matures in five years and pays annual coupons at a coupon rate of 6%.**

At issuance, the corporation will sell the bonds to investors in exchange for cash. Then, the corporation will make payments of $6 million to the bondholders at the end of each year with an additional payment of $100 million at the end of the fifth year.

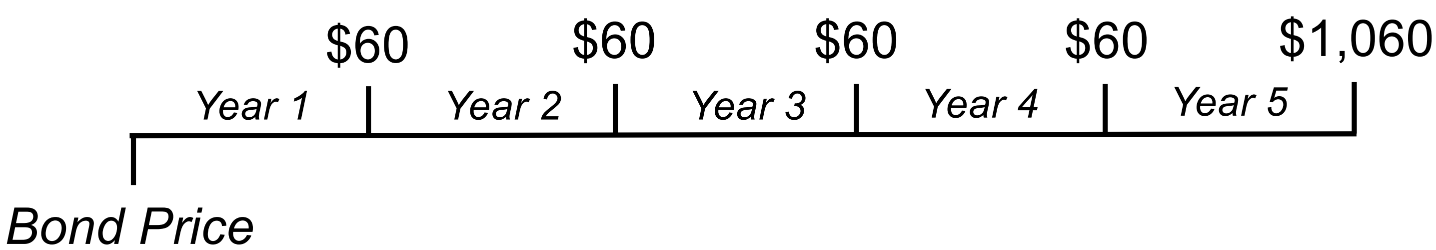
The payments are prorated, so that, for example, an investor holding $1,000 in face value of the bonds will receive $60 each year and an additional $1,000 when the bond matures:



Pricing a bond

The value, or price, of a bond is calculated as the present value of its cash flows. The discount rate used is the rate of return the investor requires on the bond.

Ex. The cash flows received for every $1,000 of the five-year corporate bond that pays annual coupon payments of 6% are:



If an investor requires an interest rate of 5% from this bond, its value is:

$$P = \frac{\$60}{1.05} + \frac{\$60}{1.05^2} + \frac{\$60}{1.05^3} + \frac{\$60}{1.05^4} + \frac{\$1{,}060}{1.05^5} = \$1{,}043.29$$

So the investor should buy this bond if it is selling for $1,043 or cheaper.

Bond prices and interest rates

If the interest rate that investors demand from a bond increase, do you think the bonds price will:

a) Increase
b) Decrease
c) Stay the same

Bond prices and interest rates

If the interest rate that investors demand from a bond increase, do you think the bonds price will:

a) Increase
b) Decrease
c) Stay the same

If the interest rate used to price the bond increases from 5% to 7%, the bond's price falls from $1,043 to $959.

Bond prices are inversely related to interest rates.

(Can you see why from the present value formula?)

Stock Prices

Stocks

- Each share of a corporation's **stock** gives the holder a proportional ownership of the company, and thus a legal claim to a proportion of the company's profits.

- In practice, corporations often reinvest a portion (or in some cases all) of their profits in a given year, but the remaining profits are distributed to shareholders as **dividends**.

Stock valuation

- Similar to a bond, a stock's value is the present value of its future cash flows, which in this case take the form of dividends.

- Unlike coupon payments on a bond, however, the dividend payments from a stock are not specified in advance and may vary with the realized profits of the company and management decisions.

$$P = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \ldots$$

- Therefore, stock valuation may require models that use simplifying assumptions.

Perpetuity formula

The present value of a stream of cash flows is calculated as:

$$PV = \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N}$$

When these **cash flows are all the same**, and when these cash flows are paid every year *through eternity*, it is known as a **perpetuity** (because the payments are perpetual), and the present value is:

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$$

Where *C* is the constant cash flow to be received at the end of each year for every year through eternity. It can be shown that, for *r* greater than zero, this can be rewritten as a simple formula, known as the **perpetuity formula**:

$$\boxed{PV = \frac{C}{r}}$$

Stock valuation: constant dividends

If we assume that the dividends paid on a stock remain constant over time, the formula for the price of the stock becomes:

$$P = \frac{D}{(1+r)} + \frac{D}{(1+r)^2} + \frac{D}{(1+r)^3} + \ldots$$

Because this is a perpetuity, the perpetuity formula may be applied. The price of the stock then reduces to:

$$P = \frac{D}{r}$$

This is known as the **constant dividend model** of stock valuation.

Valuing common stock

(Stock valuation will be considered more thoroughly later. For now, we present it as an application of the perpetuity formula.)

Ex. An investor is considering purchasing the stock of a company. The company has recently paid annual dividends of $18 per share and the investor expects the company to continue to do so. If the investor requires a 14% return on an investment in this stock, how much will he be willing to pay per share?

Ans. A corporation is usually considered to be a going concern and, for valuation purposes, is assumed to pay dividends through eternity. Since the investor is assuming a constant dividend of $18, the perpetuity formula applies, and he will value the stock at:

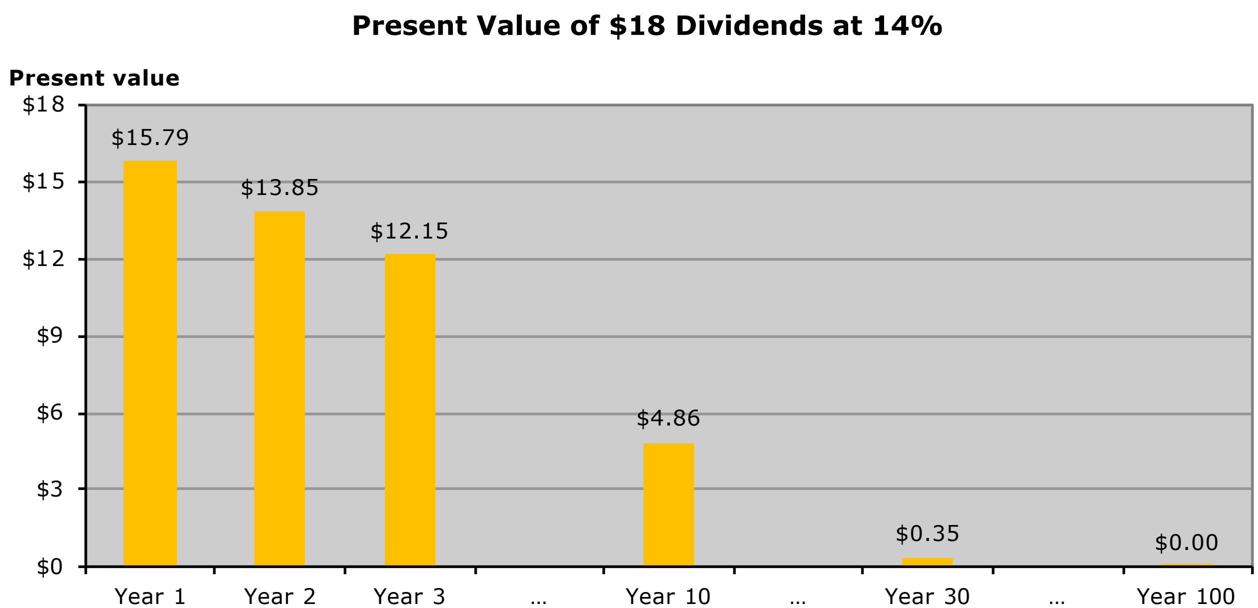
$$PV = \frac{C}{r} = \frac{\$18}{0.14} = \$128.57$$

The investor should purchase the stock if it is priced at $128.57 per share or less.

Convergence of a perpetuity

Because of the time value of money, the distant payments from a perpetuity are worth very little.

- This allows the present value of an infinite number of cash flows to be finite.
- It also makes the assumption that a corporation pays dividends forever acceptable. Even though corporations don't survive into eternity, the distant payments have little impact on the price.



Business Valuation: NPV and IRR

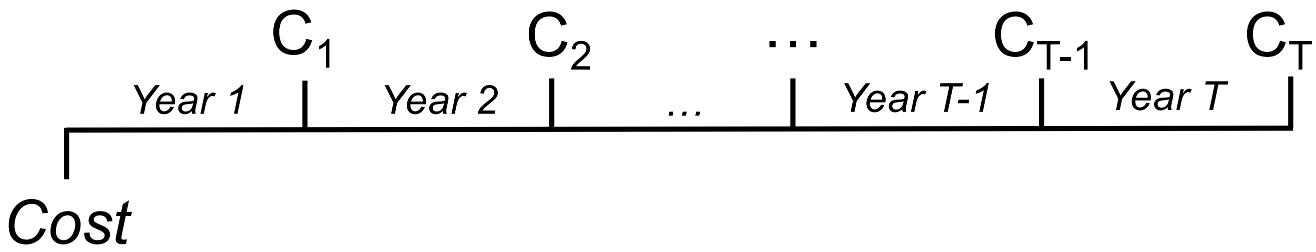
Net present value (NPV)

The net present value (NPV) is a popular method for valuing business investments.

- To calculate an NPV, calculate the present value of the cash flows generated by an investment, and subtract today's cost.
- If the NPV is greater than zero, present value of the future cash flows exceed the cost, and the investment should be undertaken.
- When the NPV is less than zero, the investment should **not** be undertaken.

Net present value (NPV)

A business project that requires an initial investment will have the cash flows:



The net present value (NPV) of this project is its present value *net* of its cost:

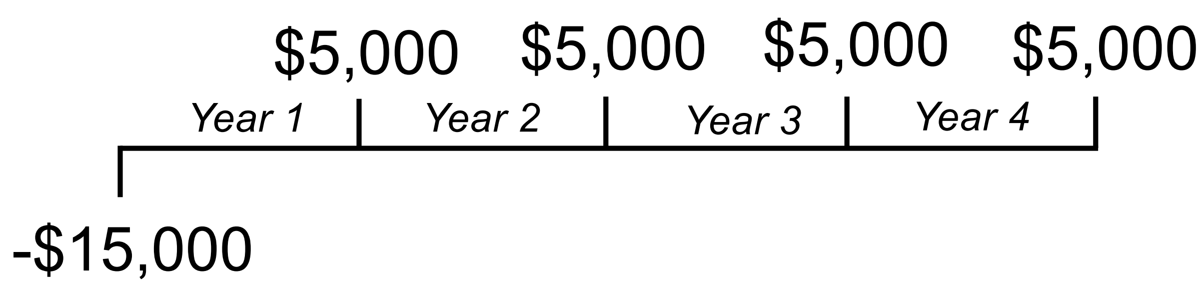
$$NPV = PV - Cost = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T} - Cost$$

NPV of two business projects

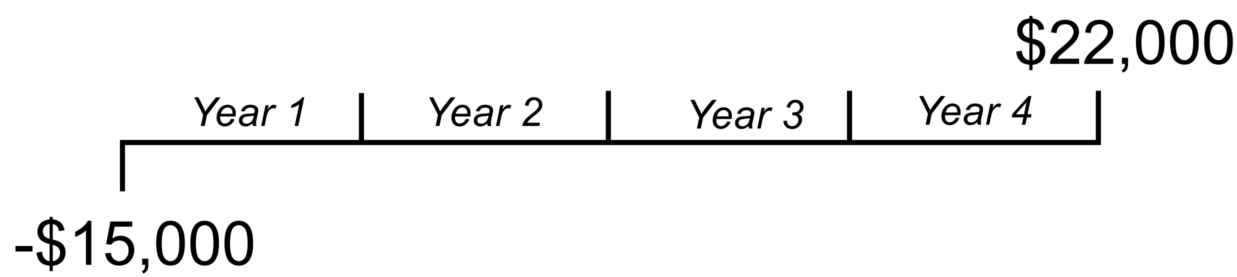
Ex. Consider two business projects, each of which require an initial investment of \$15,000 and each of which are valued with an interest rate of 11%. The first project generates \$5,000 cash flows in years one to four and the second project generates a single cash flow of \$22,000 in four years. Which project, if any, should be undertaken for \$15,000?

Ans. The cash flows for the two business projects are:

Project 1:



Project 2:



NPV of two business projects

Ans (continued). The NPV of each project is:

Project 1: $$NPV_1 = \frac{\$5{,}000}{1.11} + \frac{\$5{,}000}{1.11^2} + \frac{\$5{,}000}{1.11^3} + \frac{\$5{,}000}{1.11^4} - 15{,}000 = \$512$$

Project 2: $$NPV_2 = \frac{\$22{,}000}{1.11^4} - \$15{,}000 = -\$508$$

Since the first project has a greater NPV than project two and it's NPV is positive, project one should be undertaken.

Project 2 actually destroys value!

Note that without considering the time value of money, project 2 *looks* as if it generates greater total cash flows ($22,000) than project 1 ($20,000). Without considering the time value of money, an entrepreneur may invest in a project that actually destroys value!

Internal rate of return (IRR)

A complementary method for valuing an investment is to calculate the internal rate of return (IRR).

- The IRR is the **average return on investment**.
- It is calculated by finding the interest rate that makes the present value of an investment equal to its cost.
- In other words, it is **the interest rate that makes the NPV of an investment zero**.
- An investment should be undertaken if the IRR exceeds the benchmark return required by the investor.
- When used correctly, the IRR will lead to the same conclusion as the NPV.

Internal rate of return (IRR)

The internal rate of return must satisfy the formula:

$$Cost = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

Or, where is the cash flow today (negative for a cost):

$$0 = CF_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

The *IRR* is calculated such that it satisfies the equation. Often, the IRR cannot be solved for analytically and **must be calculated with a financial calculator**.

IRR of a business project

Ex. An entrepreneur is deciding whether to expand her business. Doing so would require an investment of $30,000 and would generate an extra profit of $20,000 a year for the next four years. Calculate the IRR for this investment. If the entrepreneur requires that the return on the investment exceed 20%, will she undertake this investment?

Ans. The IRR is calculated by solving the following formula for *IRR*:

$$0 = -\$30{,}000 + \frac{\$20{,}000}{1+IRR} + \frac{\$20{,}000}{(1+IRR)^2} + \frac{\$20{,}000}{(1+IRR)^3} + \frac{\$20{,}000}{(1+IRR)^4}$$

Using a financial calculator, we can find the solution to be:

$$IRR = 55.17\%$$

The expected return on the project exceeds 20% and so the entrepreneur would undertake the expansion.

Valuing an MBA

Financial value of an MBA

The financial value of higher education may be evaluated using the net present value methodology.

- One must compare the costs of pursuing an education to the **incremental** increases in future income.
- It's important to include not only tuition but **foregone income** sacrificed during the years of schooling.

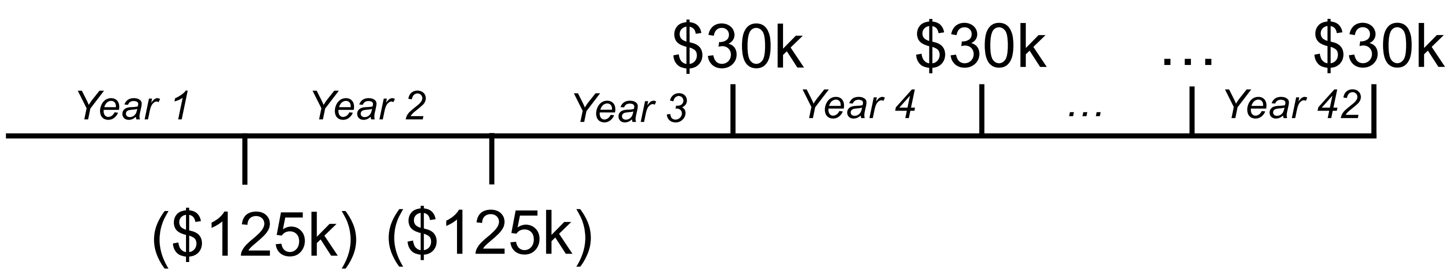
Tuition

Degree

Financial value of an MBA

Ex. Consider a student pursing a two-year MBA with a tuition of $75,000 per year. Without the MBA, the student would expect to earn around $50,000 per year for the two years spent in school and for 40 years thereafter until retirement. With the MBA, the student would earn nothing while in school, but would expect to earn around $80,000 per year – an increase of $30,000 per year – for the 40 years after graduation and until retirement.

The incremental cash flows of pursuing the MBA are:



Note that the cost is $125k because it includes the $50k of foregone income and *not* just the $75k tuition.

Financial value of an MBA

Ex. (continued)

There are 42 cash flows in this problem, so calculating the NPV by hand will take all day…

We can solve this problem easily with a **financial calculator**.

Assuming a discount rate of 8%, the **NPV of the MBA is $83,795**. With these inputs, the return on an MBA is positive and substantial!

But let's consider several scenarios…

Financial value of an MBA

Ex. (continued)

Such an analysis is sensitive to the inputs, about which there exists some uncertainty. Therefore, it's helpful to consider a range of cases. The following table assumes the same $75,000 tuition and $50,000 base salary, and shows the net present value for different discount rates and post-MBA salaries:

	Post-MBA Salary				
Discount rate	$60,000	$70,000	$80,000	$100,000	$150,000
3%	($21,305)	$196,574	$414,453	$850,211	$1,939,605
4%	($52,767)	$130,229	$313,224	$679,215	$1,594,191
5%	($76,788)	$78,850	$234,488	$545,764	$1,323,953
6%	($95,263)	$38,649	$172,560	$440,383	$1,109,941
7%	($109,558)	$6,886	$123,331	$356,219	$938,441
8%	($120,674)	($18,439)	$83,795	$288,264	$799,435
9%	($129,346)	($38,804)	$51,739	$232,824	$685,537
10%	($136,124)	($55,305)	$25,514	$187,151	$591,244

Returns on Social Security

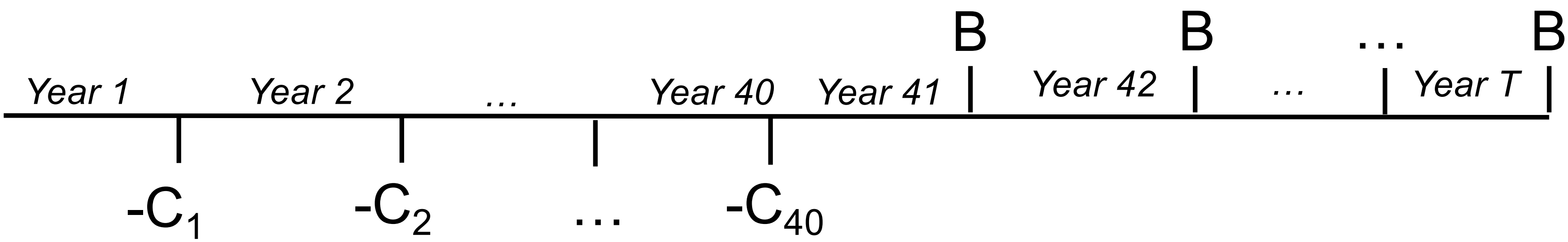
Returns on Social Security

Social Security is not an investment, per se, but the program does exhibit characteristics similar to an investment: an individual pays into the system for a number of years until retirement, at which point the individual begins collecting payments from the Social Security Administration.

The IRR methodology may be used to calculate the implied “return” on a worker’s Social Security contributions.

Returns on Social Security

Assume a Social Security participant works for 40 years between the ages 25-65 and makes annual contributions equal to 12.4% of annual income. The beneficiary then collects Social Security benefits throughout retirement. Assuming contributions of C_t at time *t* and fixed benefit payments of *B*, the participant's cash flow diagram will be:



Assume the beneficiary lives through *T* years. The return on Social Security, *R*, must satisfy the following formula:

$$0 = \sum_{t=1}^{40} \frac{-C_t}{(1+R)^t} + \sum_{t=41}^{T} \frac{B}{(1+R)^t}$$

Returns on Social Security

Ex 1. Assume beneficiary makes an average salary of $50,000 per year, which is close to the country's median salary. In this case, the annual contribution, at the contribution rate of 12.4%, is 0.124*$50,000 = $6,200. At a salary of $50,000 a year, the beneficiary is entitled to an estimated annual benefit of approximately $22,000 (the details of benefits computation are rather complex, but simplified in this example). In this case the total benefits and returns for different longevities are:

Age of Death	Return
75	-0.48%
85	1.87%
95	2.77%
105	3.22%

(The probabilities of living to age 75, 85, 95, and 105 given an age of 25 are about 70%, 40%, 8% and 0.2%, respectively.)

Bond prices and interest rates

If interest rates rise, what will typically happen to bonds prices?

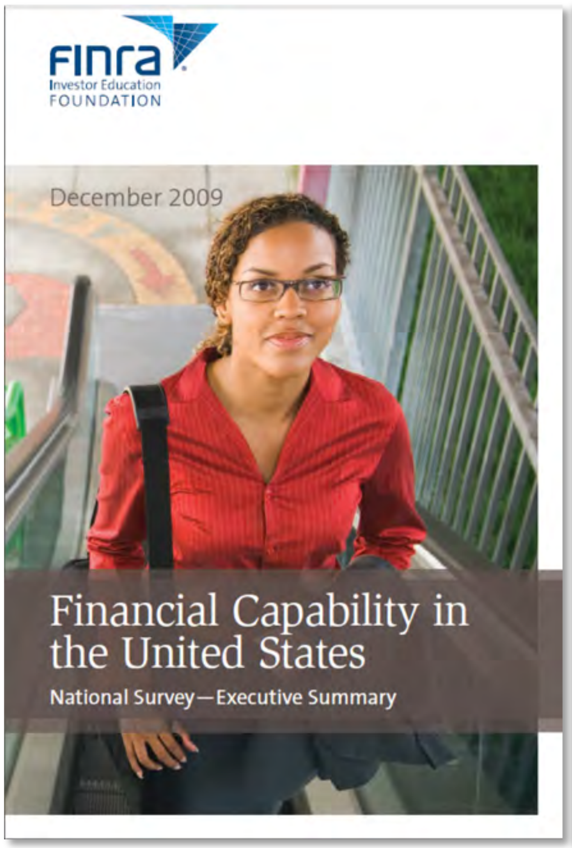
a) They will rise
b) They will fall
c) They will stay the same
d) There is not relationship between bond prices and interest rates
e) Do not know
f) Prefer not to say

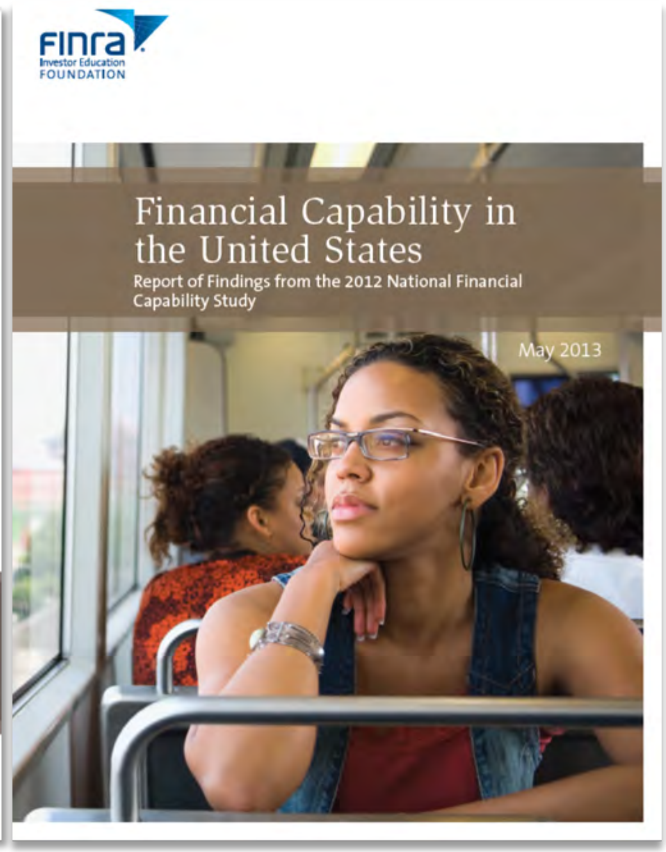
Source: 2015 National Financial Capability Survey

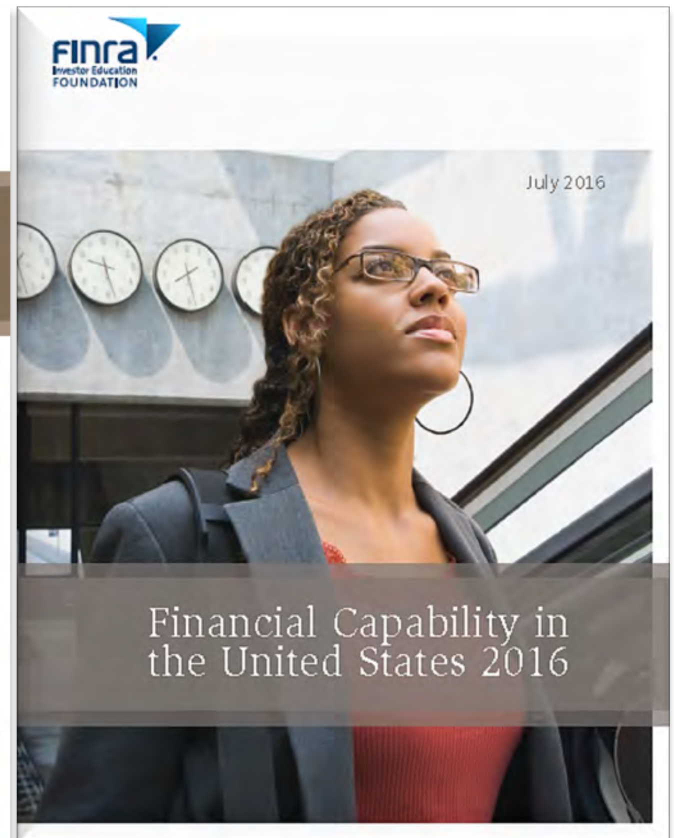
A Look at the Market/Market Opportunities

Data on financial capability

National Financial Capability Study (NFCS)







2009, 2012, and 2015 Reports by FINRA Investor Education Foundation

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b) They will fall
c) They will stay the same
d) There is not relationship between bond prices and interest rates
e) Do not know
f) Prefer not to say

27.6% answered correctly in 2009
28.1% answered correctly in 2012
28.1% answered correctly in 2015

Source: National Financial Capability Survey

Today we learned…

- ✓ Time value of money
- ✓ Present value (PV)
- ✓ Bond prices
- ✓ Stock prices
- ✓ How to use net present value (NPV) and internal rate of return (IRR)
  - ✓ Business applications
  - ✓ Valuing an MBA
  - ✓ Returns on Social Security