

## Practice Quiz 8: Personal Taxes (Solutions)

1. It is the year 2018. Ernie earned \$65,000 during that year. He is subject to the following marginal tax rates:

Income	Marginal Tax Rates
\$0 - \$9,525	10%
\$9,526 - \$38,700	12%
\$38,701 - \$82,500	22%
\$82,501 - \$157,500	24%
\$157,501 - \$200,000	32%
\$200,001 - \$500,000	35%
Over \$500,000	37%

Ernie qualifies for a \$12,000 standard deduction. In 2018, he paid \$2,500 in student loan interest and made a \$2,000 charitable contribution to his alma mater, both of which are tax deductible expenses. Note that the \$2,500 student loan interest is an above-the-line deduction, meaning he can claim this deduction to obtain the adjusted gross income, but he cannot claim the charitable contribution deduction and standard deduction, as the charitable deduction is a below-the-line deduction. He can choose to itemize the charitable deduction or claim the standard deduction. How much must Ernie pay in taxes in 2018? What is his effective tax rate?

**Ans.** Because the standard deduction plus the student loan interest deduction is greater than his \$2,000 in itemizable deductions, Ernie may reduce his taxable income by the \$12,000 standard deduction and \$2,500 student loan interest deduction. Ernie's taxable income is therefore:

$$\$65,000 - \$14,500 = \$50,500$$

His taxes owed are calculated as:

$$Taxes = 0.10 * \$9,525 + 0.12 * (\$38,700 - \$9,525) + 0.22 * (\$50,500 - \$38,700) = \$7,049.50$$

His effective tax rate is then:

$$Effective\ Tax\ Rate = \frac{\$7,049.50}{\$65,000} = 10.85\%$$

**2.** At the beginning of the year, Mr. and Mrs. Jacobson have a current mortgage balance of \$120,000, on which they pay a 6% APR. They itemize their deductions, and would do so whether or not they had mortgage. They are subject to a tax rate of 24%. How much will the mortgage interest deduction save the Jacobsons during the year? What is the after-tax interest rate on their mortgage? (For simplicity, assume the mortgage payment is made annually.)

**Ans.** The Jacobsons will pay  $0.06 * \$120,000 = \$7,200$  in mortgage interest at the end of the year. The mortgage interest deduction allows them to reduce their taxable income by this amount, allowing them to save  $0.24 * \$7,200 = \$1,728$  in reduced taxes.

The after-tax interest rate on their mortgage is  $6\% * (1 - 0.24) = 4.56\%$ .

**3.** Amy and Joe both invest \$5,000 in the stock market. Amy invests passively and holds onto her stocks for 30 years. Joe actively trades, turning over his portfolio yearly. Consequently, Amy is subjected to a deferred long-term capital gains tax of 15%, while Joe is subject to an annual short-term capital gains tax at his marginal income tax rate, which is 32%. If both earn an annual return of 8% on their investment, how much will each have in 30 years, after tax? What will be their after-tax returns? What pre-tax return must Joe achieve through active trading to match Amy's wealth in 30 years?

**Ans.** Because Amy invests for the long-term and is subject to a deferred capital gains tax of 15%, her final wealth will grow to:

$$F = P * [(1 + r)^N (1 - T) + T] = \$5,000[(1.08)^{30}(1 - 0.15) + 0.15] = \$43,516$$

And her after-tax return will be:

$$r_{after-tax} = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 = \left(\frac{\$43,516}{\$5,000}\right)^{\frac{1}{30}} - 1 = 7.48\%$$

Because Joe trades actively and turns over his portfolio every year, he is subject to a 30% accrual tax:

$$F = P[1 + r(1 - T)]^N = \$5,000[1 + .08(1 - 0.32)]^{30} = \$24,498.1$$

And his after-tax return will be:

$$r_{after-tax} = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 = \left(\frac{\$24,498}{\$5,000}\right)^{\frac{1}{30}} - 1 = 5.44\%$$

Amy achieves a much higher ending wealth than Joe because she is taxed at a lower effective rate. However, because Joe is trading actively, he may be able to achieve better portfolio performance than Amy through savvy stock selection. To at least match Amy's ending wealth, Joe's return must satisfy the following:

$$F = P[1 + r(1 - T)]^N = \$43,516$$

$$\rightarrow r = \frac{1}{1 - T} \left[ \left( \frac{\$43,516}{P} \right)^{\frac{1}{N}} - 1 \right] = \frac{1}{1 - 0.32} \left[ \left( \frac{\$43,516}{\$5,000} \right)^{\frac{1}{30}} - 1 \right] = 11\%$$

In other words, Joe must beat the market by 3% each year for 30 years to justify his active trading. If markets are efficient, this will be difficult to do!

**4.** Jason wishes to invest \$3,000 (in *after-tax* dollars) for retirement. He will do so in a stock index fund and expects an average annual return of 7%. Compare the after-tax value of Jason's contribution in 30 years if he (a) invests outside of any tax-advantaged account, (b) in a Roth IRA, and (c) in a traditional IRA. Assume a long-term capital gains tax of 15% and an income tax rate of 25% both today and when Jason withdraws in 30 years. Discuss how the after-tax value of an investment in a Roth IRA versus traditional IRA would change if Jason's tax rate in retirement is lower than it is today.

**Ans.** If Jason invests in a non-tax-advantaged account, he will be taxed at the long-term capital gains rate in 25 years:

$$F = P[(1 + r)^N(1 - T) + T] = \$3,000 * [1.07^{30}(1 - 0.15) + 0.15] = \$19,861$$

If Jason invests in a Roth IRA, his withdrawal will be tax exempt:

$$F = P(1 + r)^N = \$3,000 * 1.07^{30} = \$22,837$$

If Jason instead invests in a traditional IRA, he will make contributions with *pre-tax* income. The pre-tax equivalent to \$3,000 in *after-tax* dollars is \$4,000:

$$P_{after-tax} = P_{pre-tax}(1 - T) \rightarrow P_{pre-tax} = \frac{P_{after-tax}}{1 - T} = \frac{\$3,000}{1 - .25} = \$4,000$$

Taxes on such a contribution will then be deferred:

$$F = P(1 + r)^N(1 - T) = \$4,000(1.07)^{30}(1 - 0.25) = \$22,837$$

When the tax rate is the same today and at retirement, the future values of a Roth IRA contribution and a traditional IRA contribution are equal. If Jason's income tax rate at retirement, however, will be lower than it is today, the after-tax value of an investment in a traditional IRA is greater than that of an investment in a Roth IRA.