

# The Time Value of Money

Lecture 2

# Essential concepts in personal finance

Video about essential concepts in personal finance

**“The ABCs of Personal Finance”**

<https://vimeo.com/176664740>

# Today's lecture

Lecture 2.1 discusses time value of money in depth.

Lecture 2.2 explains how to use a financial calculator.

- Optional lecture

# Time Value of Money

## Lecture 2.1



# \$100K today or in a year?

Would you rather receive \$100,000 today or \$100,000 one year from now?

Today



\$100,000

**OR**

One Year



\$100,000

- a) \$100,000 today
- b) \$100,000 in one year
- c) It's the same amount, so it doesn't matter
- d) It depends

# \$100K today or in a year?

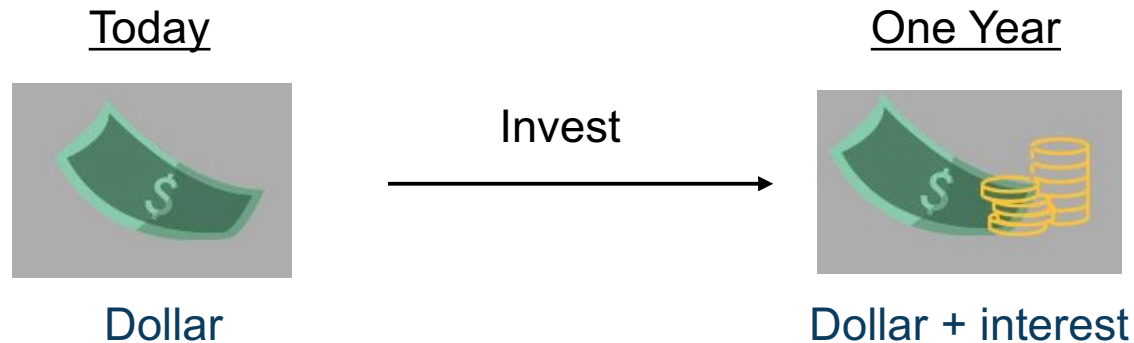
You should prefer \$100,000 today.



- a) **\$100,000 today**
- b) \$100,000 in one year
- c) It's the same amount, so it doesn't matter
- d) It depends

# \$100K today or in a year?

This is because a dollar today can be invested and earns interest.



- It's better to have a dollar today than in the future. You can invest it and get more than a dollar in the future.
- **A dollar today is worth more than a dollar in the future.**

# The time value of money

**A dollar today is more valuable than a dollar later.**

**This is called the time value of money...**





# \$100K today or \$110K in a year?

Let's do a trickier one: would you rather receive \$100,000 today or \$110,000 one year from now?

Today



\$100,000

**OR**

One Year



\$110,000

- a) \$100,000 today, because I need money today
- b) \$110,000 in one year because it's more
- c) It depends

# \$100K today or \$110K in a year?

**“It depends”**

It depends on the interest rate!

If the interest rate is **5%**, it would be better to **take \$110,000 in one year**, because \$100,000 will not grow to \$110,000, even after interest.

$$\$100,000 * (1 + r) = \$100,000 * 1.05 = \$105,000$$

- But if the interest rate is **20%**, it would be better to **take the \$100,000 today**, because it will grow to more than \$110,000 after one year.

$$\$100,000 * (1 + r) = 100,000 * 1.20 = \$120,000$$

# \$100K today or \$110K in a year?

“What if I need the money today?”

Then borrow!

- If the interest rate is **5%**, still **take \$110,000 in one year**, but borrow \$100,000 today.
- After one year, you will owe \$105,000.
- You can use the \$110,000 in one year to repay the \$105,000 and still have \$5,000 left over.
- You get your \$100,000 today and also \$5,000 in one year!
- But if the interest rate is **20%**, **take the \$100,000 today**. If you borrow, you will owe more than \$110,000 in one year.

# \$100K today or \$110K in a year?

So, whether \$100K today is more valuable than \$110K in one year depends on the interest rate.

Today



\$100,000

One Year



\$110,000

@ 5%

If the interest rate is 5%, \$110K in a year is worth more.

Today



\$100,000

One Year



\$110,000

@ 20%

If the interest rate is 20%, \$100K today is worth more.

# \$100K today or \$110K in a year?

“What if the interest rate is 10%?”

- At 10% interest, \$100K today will grow to \$110K in a year:

$$\$100,000 * 1.10 = \$110,000$$

- So, if the interest rate is **10%**, you should be indifferent between the two options: **\$110K in one year is exactly worth \$100K today.**



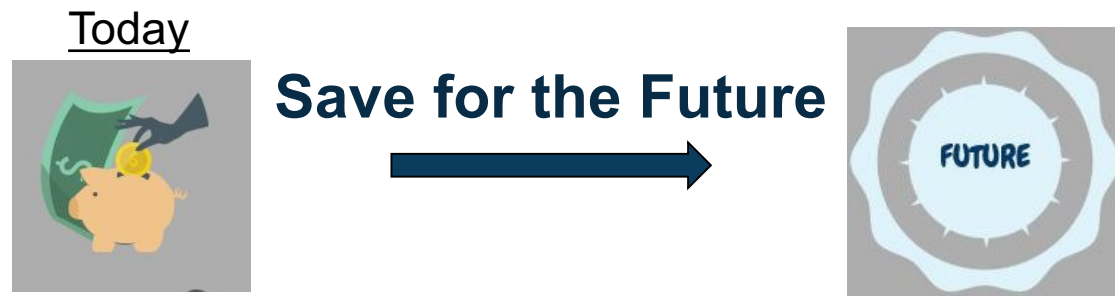
# Present Value



# Moving money through time

Consumers can “move” money across time by savings and borrowing.

- Consumers often move their money into the future, for example by saving for retirement.



- Consumers also use their future income today by taking out loans, for example to purchase a home today.



# Moving money through time

- But as we saw earlier, the value of money changes at different points in time.



- So we need a way to compare the value of cash flows received at different times...
- To do so, we will introduce the concept of **present value**.



# Present value

The **present value** of a future cash flow is the value of that cash flow in today's dollars.

Earlier, we found that \$110,000 in one year is worth \$100,000 today when the interest rate is 10%. So, in finance, we say:

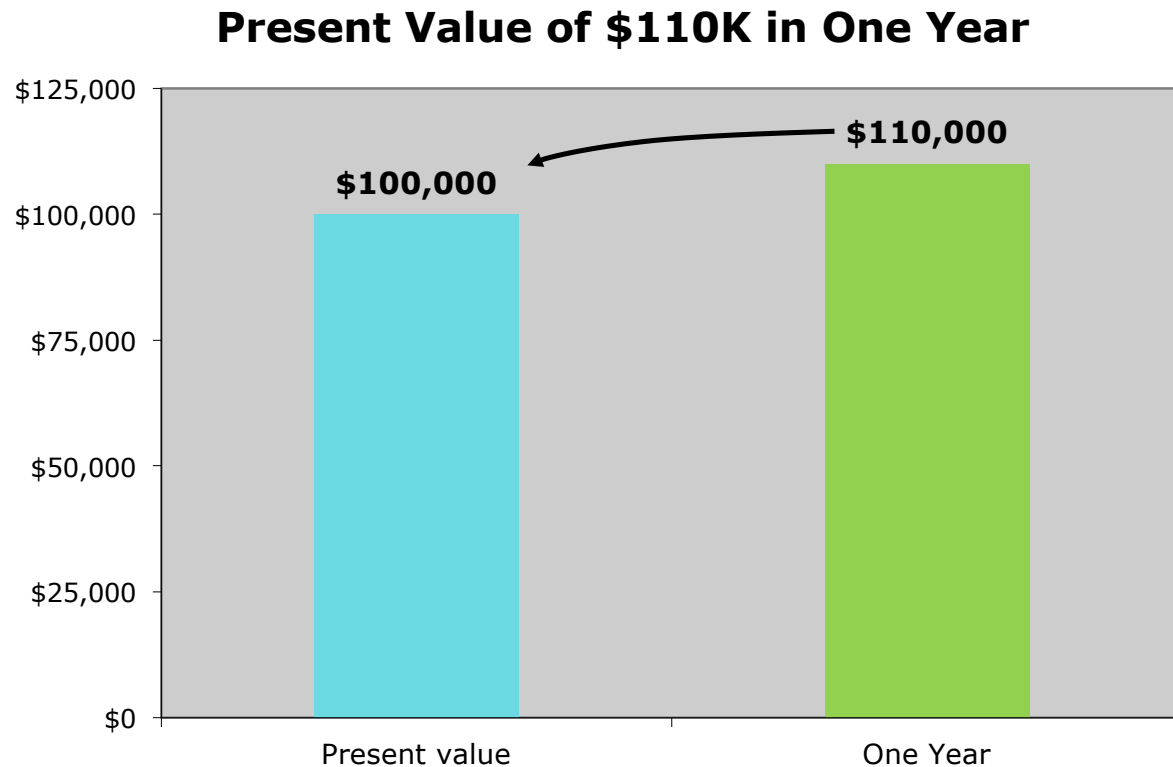
The **present value** of \$110,000 in one year is \$100,000.  
(when the interest rate is 10%)

Notice that \$110,000 to be received in the future is worth less than its face value today.

This is generally true, and the further in the future the \$110,000 will be received, the less it will be worth today. Once again, this is the **time value of money**...

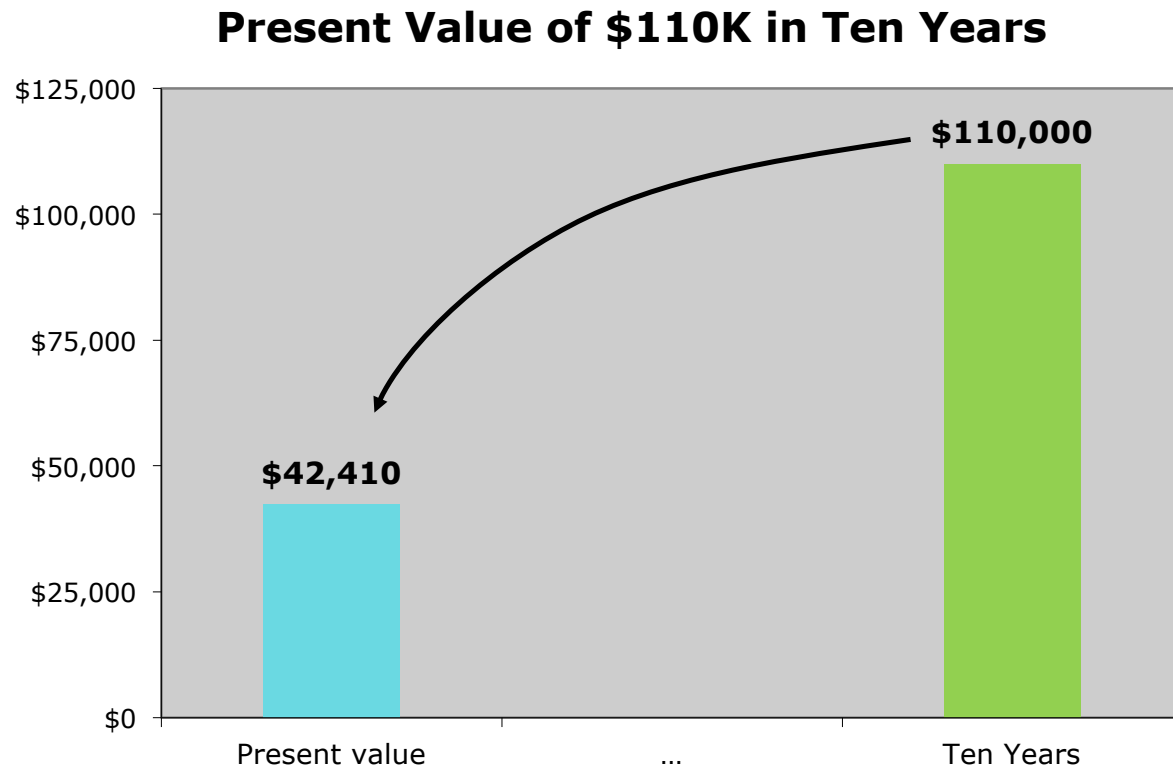
# Visualizing the time value of money

\$110K to be received in one year is worth \$100K today if the interest rate is 10%.



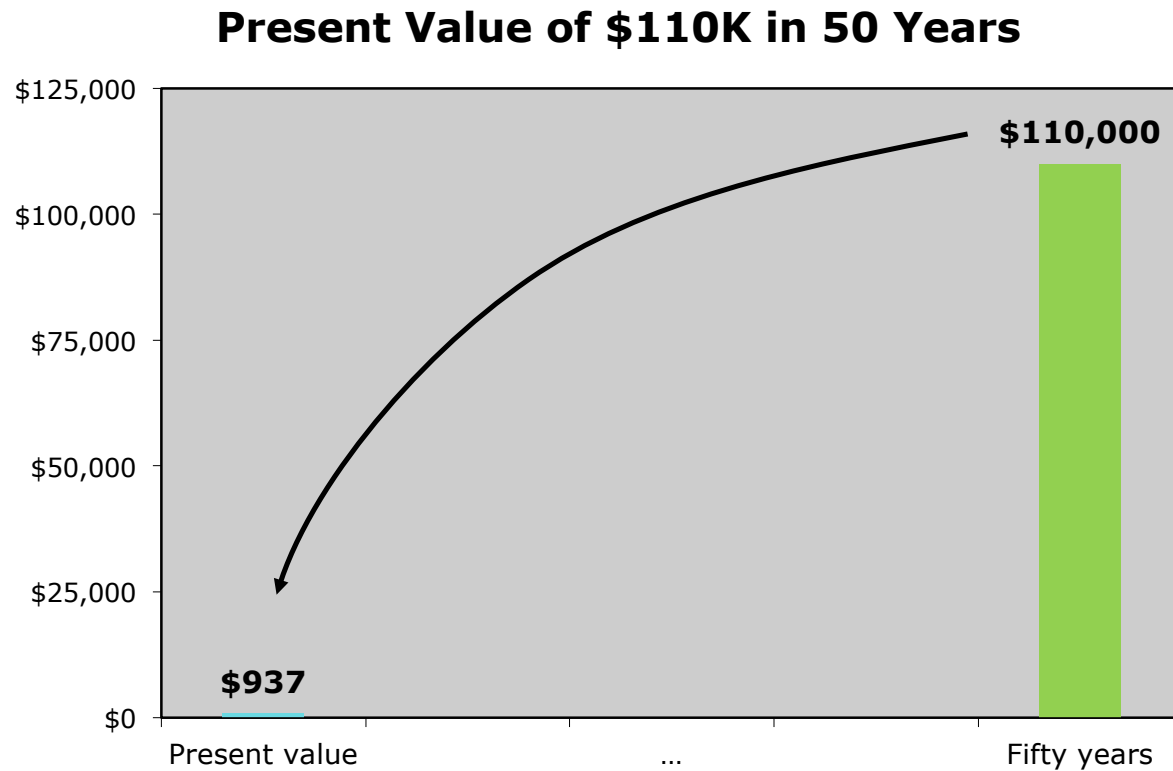
# Visualizing the time value of money

But worth only \$42,000 if it will be received in *ten years*...



# Visualizing the time value of money

And worth only \$900 if it will be received in 50 years!



# Calculating a present value

Let's see how to calculate a present value.

The present value of \$110,000 to be received in ten years is the amount that will grow to \$110,000 if it is invested today. This must satisfy the interest compounding formula:

$$F = P(1 + r)^T$$

Where the ending value is \$110,000, the interest rate is 10%, and the savings compound for ten years:

$$\$110,000 = P(1.10)^{10}$$

Dividing both sides by  $(1.10)^{10}$ , we can find the present value:

$$P = \frac{\$110,000}{(1.10)^{10}} = \$42,410$$

# The present value formula

In general, the present value  $PV$  can be found by rearranging the interest compounding formula:

$$F = PV(1 + r)^T$$



$$PV = \frac{F}{(1 + r)^T}$$

**Where:**

**PV = present value**

**F = future cash flow received at time T**

**r = interest rate**

# Discounting and the time value of money

Computing the present value is called **discounting**, and it is the opposite of **interest compounding**.

From the present value formula we see that the **time value of money** is the power of interest compounding in reverse!

$$PV = \frac{F}{(1 + r)^T}$$

As a dollar is pushed further into the future,  $T$  becomes larger, and its present value becomes smaller.

And the higher the interest rate  $r$ , the more quickly it shrinks.

# Valuing a lottery payout

Many state lotteries require the winner to receive payments spread into the future to collect the full jackpot. If the winner wants the cash today, he or she may only receive a fraction of the winnings.

**Ex.** Michael wins the state lottery with a \$250,000 jackpot. To receive the full jackpot, he must wait five years. If he collects his winnings today, he will only receive \$200,000. If the interest rate is 6%, what is the better deal?

**Ans.** The present value of the \$250,000 jackpot in five years can be found by discounting the sum by 6%:

$$PV = \frac{F}{(1 + r)^T} = \frac{\$250,000}{1.06^5} = \$186,815$$

Because this is less than \$200,000, it is better to take the \$200,000 today.





# Present value of a stream of cash flows

In many situations, there are multiple cash flows to value.



The present value of a stream of cash flows is the sum of the present values of the individual cash flows:

$$PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

Where  $C_T$  is the cash flow at time  $T$ .

# Applications of present value

Situations where you might have to find the present value of a stream of cash flows include:

- **Establishing a business:** A good business will generate income each year, but will usually require an up-front investment. To determine whether the business is worth starting, the present value of the income it generates should be compared to the cost of the investment today.
- **Pursuing education:** A good education will increase a graduate's earnings over a lifetime. To determine if a degree is financially valuable, compare the tuition cost to the present value of the increase in future earnings.
- **Valuing financial assets:** A financial asset, such as a stock or bond, is a contract that promises payments to the holder. To determine if an asset is worth buying, compare its price to the present value of its promised payments.

# Pricing a Bond



# What is a bond?

A bond is an example of financial asset.

- A bond may be issued by a **corporation or government**, and effectively acts as a loan to the issuer.
- For long-term bonds, the issuer issues the security to investors in exchange for cash. The issuer then pays periodic interest on the bond in installments known as **coupon payments** and, at the bond's maturity, the issuer also pays back the original **face value** of the bond.



# THE UNITED STATES OF AMERICA

FOR VALUE RECEIVED PROMISES TO PAY TO



-THE FIRST NATIONAL BANK OF BOSTON,  
BOSTON, MASSACHUSETTS-

1

2½%

**TREASURY BOND  
of 1963**

Dated December 15, 1954

Due August 15, 1963

OR REGISTERED ASSIGNS THE SUM OF  
**ONE HUNDRED THOUSAND DOLLARS**

ON AUGUST 15, 1963, AND TO PAY INTEREST ON SAID PRINCIPAL SUM FROM DEC 15 1954, AT THE RATE OF TWO AND ONE-HALF PERCENT PER ANNUM, PAYABLE ON A SEMIANNUAL BASIS ON AUGUST 15, 1955, AND THEREAFTER ON FEBRUARY 15 AND AUGUST 15 IN EACH YEAR UNTIL THE PRINCIPAL HEREOF SHALL BE PAYABLE, AT THE TREASURY DEPARTMENT, WASHINGTON, D. C., OR, AT THE HOLDER'S OPTION, AT ANY AGENCY OR AGENCIES IN THE UNITED STATES WHICH THE SECRETARY OF THE TREASURY MAY FROM TIME TO TIME DESIGNATE FOR THE PURPOSE. THIS BOND IS ONE OF A SERIES OF BONDS OF THE UNITED STATES, AUTHORIZED BY THE SECOND LIBERTY BOND ACT, AS AMENDED, ISSUED PURSUANT TO AND SUBJECT TO THE TERMS AND CONDITIONS OF TREASURY DEPARTMENT CIRCULAR NO. 982, DATED NOVEMBER 22, 1954, AND DESIGNATED 2½ PERCENT TREASURY BONDS OF 1963. THIS BOND IS NOT SUBJECT TO CALL FOR REDEMPTION PRIOR TO MATURITY. THE INCOME DERIVED FROM THIS BOND SHALL BE SUBJECT TO ALL TAXES NOW OR HEREAFTER IMPOSED UNDER THE INTERNAL REVENUE CODE, OR LAWS AMENDATORY OR SUPPLEMENTARY THERETO. THIS BOND SHALL BE SUBJECT TO ESTATE, INHERITANCE, GIFT OR OTHER EXCISE TAXES, WHETHER FEDERAL OR STATE, BUT SHALL BE EXEMPT FROM ALL TAXATION NOW OR HEREAFTER IMPOSED ON THE PRINCIPAL OR INTEREST HEREOF BY ANY STATE, OR ANY OF THE POSSESSIONS OF THE UNITED STATES, OR BY ANY LOCAL TAXING AUTHORITY. THIS BOND IS ACCEPTABLE TO SECURE DEPOSITS OF PUBLIC MONIES. THIS BOND IS SUBJECT TO THE GENERAL REGULATIONS OF THE TREASURY DEPARTMENT WITH RESPECT TO UNITED STATES BONDS.

WASHINGTON, D. C., DECEMBER 15, 1954.

*William F. Humphrey*  
SECRETARY OF THE TREASURY

TRANSFERABLE ON THE BOOKS OF THE UNITED STATES TREASURY DEPARTMENT.

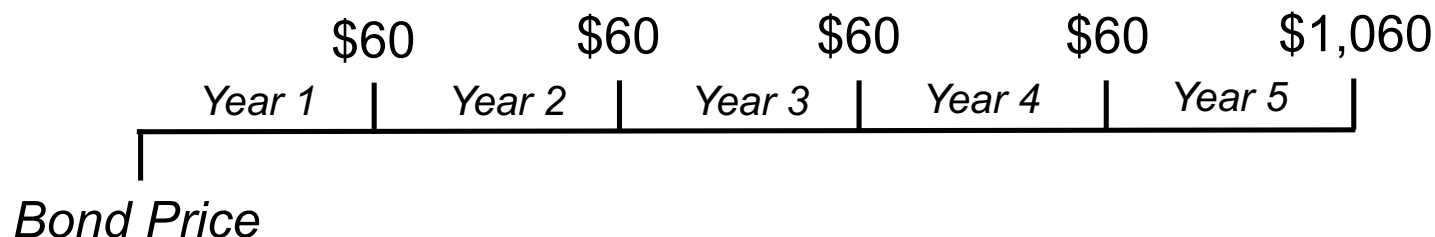
**ONE HUNDRED THOUSAND DOLLARS**

# What is a bond?

**Ex.** A corporation issues a bond with a **face value of \$100 million that matures in five years and pays annual coupons at a coupon rate of 6%.**

At issuance, the corporation will sell the bonds to investors in exchange for cash. Then, the corporation will make payments of \$6 million to the bondholders at the end of each year with an additional payment of \$100 million at the end of the fifth year.

The payments are prorated, so that, for example, an investor holding \$1,000 in face value of the bonds will receive \$60 each year and an additional \$1,000 when the bond matures:

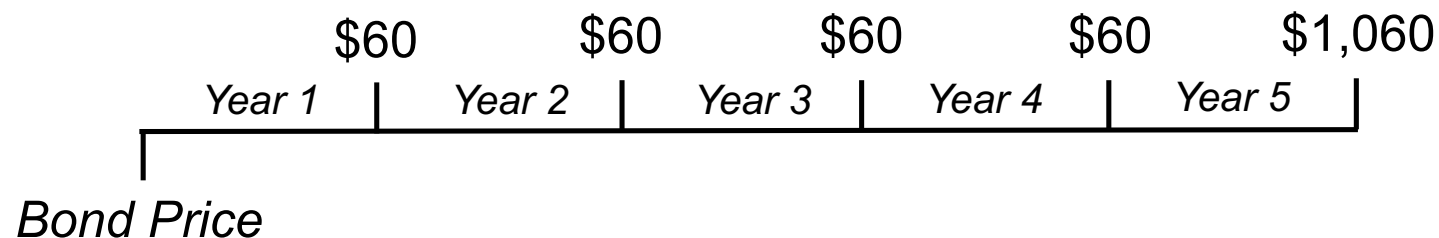




# Pricing a bond

The value, or price, of a bond is calculated as the present value of its cash flows. The discount rate used is the rate of return the investor requires on the bond.

**Ex.** The cash flows received for every \$1,000 of the five-year corporate bond that pays annual coupon payments of 6% are:



If an investor requires an interest rate of 5% from this bond, its value is:

$$P = \frac{\$60}{1.05} + \frac{\$60}{1.05^2} + \frac{\$60}{1.05^3} + \frac{\$60}{1.05^4} + \frac{\$1,060}{1.05^5} = \$1,043.29$$

So the investor should buy this bond if it is selling for \$1,043 or cheaper.

# Bond prices and interest rates

If the interest rate that investors demand from a bond increase, do you think the bonds price will:

- a) Increase
- b) Decrease
- c) Stay the same



# Bond prices and interest rates

If the interest rate that investors demand from a bond increase, do you think the bonds price will:

- a) Increase
- b) Decrease**
- c) Stay the same

If the interest rate used to price the bond increases from 5% to 7%, the bond's price falls from \$1,043 to \$959.

**Bond prices are inversely related to interest rates.**

(Can you see why from the present value formula?)

# Stock Prices



# Stocks

- Each share of a corporation's **stock** gives the holder a proportional ownership of the company, and thus a legal claim to a proportion of the company's profits.
- In practice, corporations often reinvest a portion (or in some cases all) of their profits in a given year, but the remaining profits are distributed to shareholders as **dividends**.

# Stock valuation

- Similar to a bond, a stock's value is the present value of its future cash flows, which in this case take the form of dividends.
- Unlike coupon payments on a bond, however, the dividend payments from a stock are not specified in advance and may vary with the realized profits of the company and management decisions.

$$P = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \dots$$

- Therefore, stock valuation may require models that use simplifying assumptions.

# Perpetuity formula

The present value of a stream of cash flows is calculated as:

$$PV = \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_N}{(1+r)^N}$$

When these **cash flows are all the same**, and when these cash flows are paid every year *through eternity*, it is known as a **perpetuity** (because the payments are perpetual), and the present value is:

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

Where  $C$  is the constant cash flow to be received at the end of each year for every year through eternity. It can be shown that, for  $r$  greater than zero, this can be rewritten as a simple formula, known as the **perpetuity formula**:

$$PV = \frac{C}{r}$$

# Stock valuation: constant dividends

If we assume that the dividends paid on a stock remain constant over time, the formula for the price of the stock becomes:

$$P = \frac{D}{(1+r)} + \frac{D}{(1+r)^2} + \frac{D}{(1+r)^3} + \dots$$

Because this is a perpetuity, the perpetuity formula may be applied. The price of the stock then reduces to:

$$P = \frac{D}{r}$$

This is known as the **constant dividend model** of stock valuation.

# Valuing common stock

(Stock valuation will be considered more thoroughly later. For now, we present it as an application of the perpetuity formula.)

**Ex.** An investor is considering purchasing the stock of a company. The company has recently paid annual dividends of \$18 per share and the investor expects the company to continue to do so. If the investor requires a 14% return on an investment in this stock, how much will he be willing to pay per share?

**Ans.** A corporation is usually considered to be a going concern and, for valuation purposes, is assumed to pay dividends through eternity. Since the investor is assuming a constant dividend of \$18, the perpetuity formula applies, and he will value the stock at:

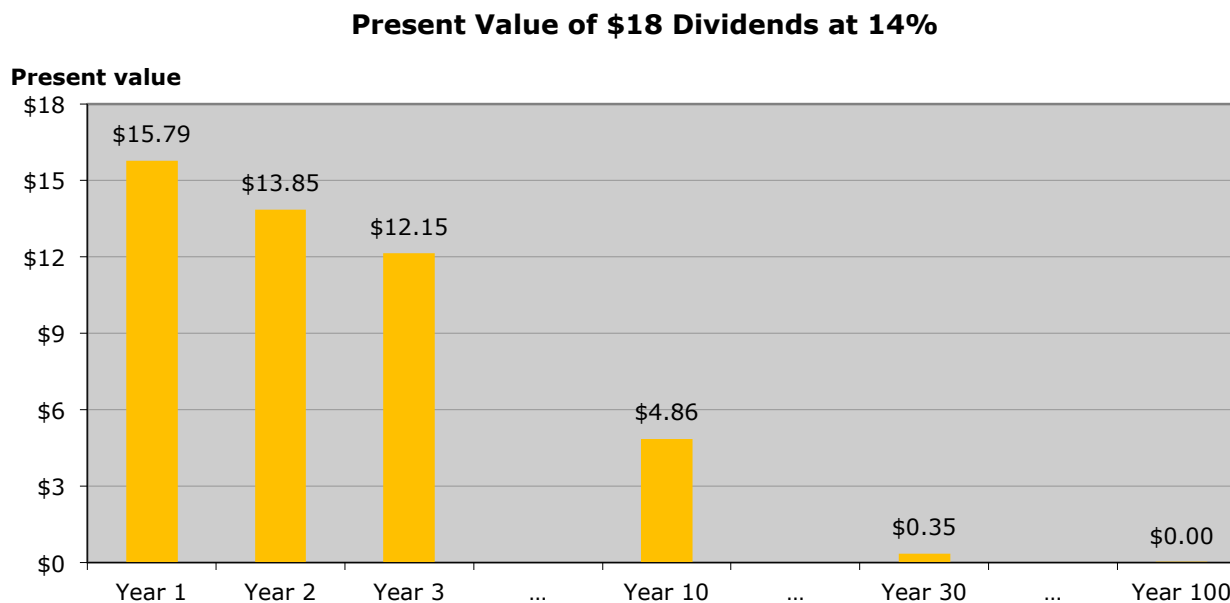
$$PV = \frac{C}{r} = \frac{\$18}{0.14} = \$128.57$$

The investor should purchase the stock if it is priced at \$128.57 per share or less.

# Convergence of a perpetuity

Because of the time value of money, the distant payments from a perpetuity are worth very little.

- This allows the present value of an infinite number of cash flows to be finite.
- It also makes the assumption that a corporation pays dividends forever acceptable. Even though corporations don't survive into eternity, the distant payments have little impact on the price.





# **Business Valuation: NPV and IRR**



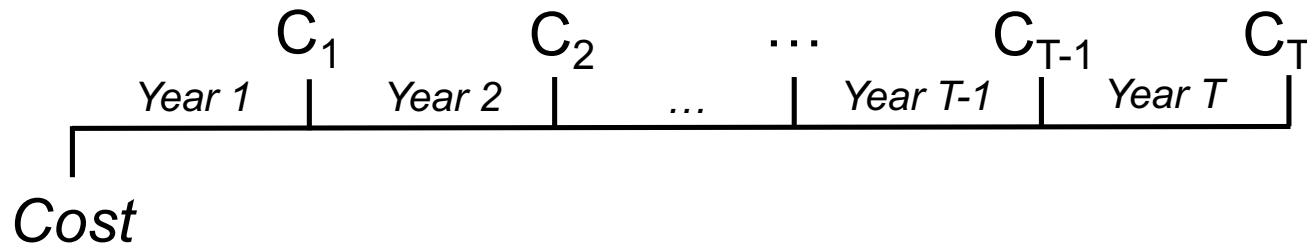
# Net present value (NPV)

The net present value (NPV) is a popular method for valuing business investments.

- To calculate an NPV, calculate the present value of the cash flows generated by an investment, and subtract today's cost.
- If the NPV is greater than zero, present value of the future cash flows exceed the cost, and the investment should be undertaken.
- When the NPV is less than zero, the investment should **not** be undertaken.

# Net present value (NPV)

A business project that requires an initial investment will have the cash flows:



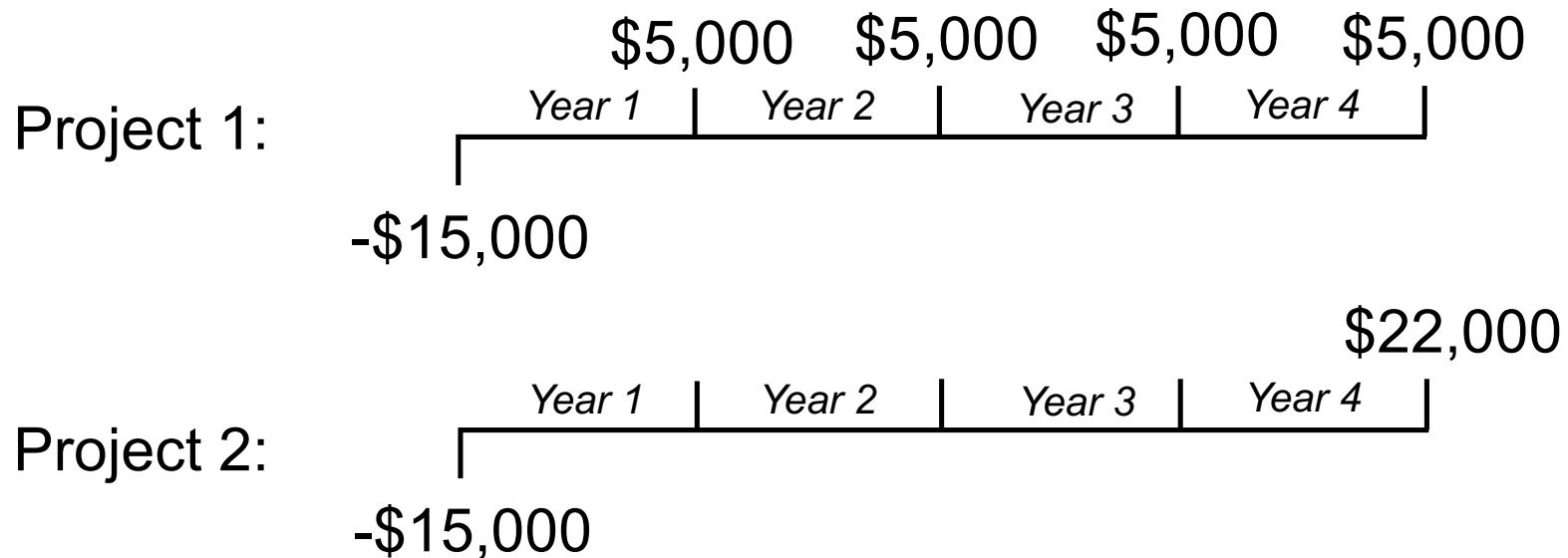
The net present value (NPV) of this project is its present value *net* of its cost:

$$NPV = PV - Cost = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T} - Cost$$

# NPV of two business projects

**Ex.** Consider two business projects, each of which require an initial investment of \$15,000 and each of which are valued with an interest rate of 11%. The first project generates \$5,000 cash flows in years one to four and the second project generates a single cash flow of \$22,000 in four years. Which project, if any, should be undertaken for \$15,000?

**Ans.** The cash flows for the two business projects are:



# NPV of two business projects

**Ans (continued).** The NPV of each project is:

$$\text{Project 1: } NPV_1 = \frac{\$5,000}{1.11} + \frac{\$5,000}{1.11^2} + \frac{\$5,000}{1.11^3} + \frac{\$5,000}{1.11^4} - 15,000 = \$512$$

$$\text{Project 2: } NPV_2 = \frac{\$22,000}{1.11^4} - \$15,000 = -\$508$$

Since the first project has a greater NPV than project two and it's NPV is positive, project one should be undertaken.

Project 2 actually destroys value!

Note that without considering the time value of money, project 2 *looks* as if it generates greater total cash flows (\$22,000) than project 1 (\$20,000). Without considering the time value of money, an entrepreneur may invest in a project that actually destroys value!

# Internal rate of return (IRR)

A complementary method for valuing an investment is to calculate the internal rate of return (IRR).

- The IRR is the **average return on investment**.
- It is calculated by finding the interest rate that makes the present value of an investment equal to its cost.
- In other words, it is **the interest rate that makes the NPV of an investment zero**.
- An investment should be undertaken if the IRR exceeds the benchmark return required by the investor.
- When used correctly, the IRR will lead to the same conclusion as the NPV.

# Internal rate of return (IRR)

The internal rate of return must satisfy the formula:

$$Cost = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

Or, where  $CF_0$  is the cash flow today (negative for a cost):

$$0 = CF_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

The *IRR* is calculated such that it satisfies the equation. Often, the IRR cannot be solved for analytically and **must be calculated with a financial calculator**.

# IRR of a business project

**Ex.** An entrepreneur is deciding whether to expand her business. Doing so would require an investment of \$30,000 and would generate an extra profit of \$20,000 a year for the next four years. Calculate the IRR for this investment. If the entrepreneur requires that the return on the investment exceed 20%, will she undertake this investment?

**Ans.** The IRR is calculated by solving the following formula for *IRR*:

$$0 = -\$30,000 + \frac{\$20,000}{1 + IRR} + \frac{\$20,000}{(1 + IRR)^2} + \frac{\$20,000}{(1 + IRR)^3} + \frac{\$20,000}{(1 + IRR)^4}$$

Using a financial calculator, we can find the solution to be:

$$IRR = 55.17\%$$

The expected return on the project exceeds 20% and so the entrepreneur would undertake the expansion.



# **Valuing an MBA**



# Financial value of an MBA

The financial value of higher education may be evaluated using the net present value methodology.

- One must compare the costs of pursuing an education to the **incremental** increases in future income.
- It's important to include not only tuition but **foregone income** sacrificed during the years of schooling.

Tuition



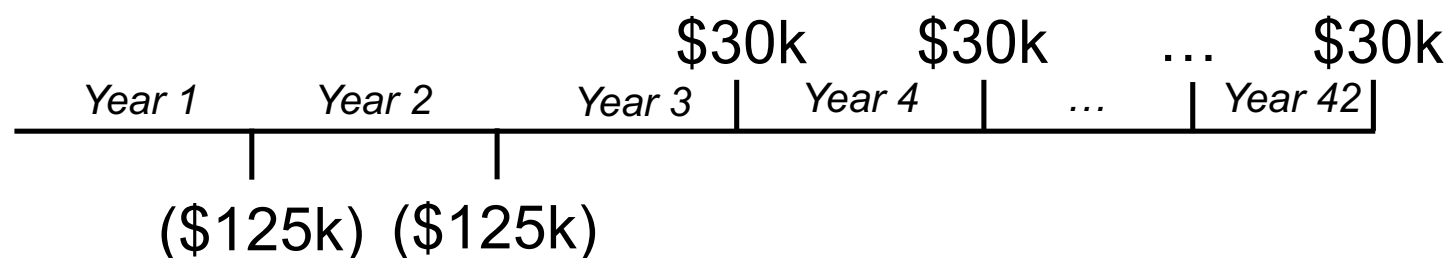
Degree



# Financial value of an MBA

**Ex.** Consider a student pursuing a two-year MBA with a tuition of \$75,000 per year. Without the MBA, the student would expect to earn around \$50,000 per year for the two years spent in school and for 40 years thereafter until retirement. With the MBA, the student would earn nothing while in school, but would expect to earn around \$80,000 per year – an increase of \$30,000 per year – for the 40 years after graduation and until retirement.

The incremental cash flows of pursuing the MBA are:



Note that the cost is \$125k because it includes the \$50k of foregone income and *not* just the \$75k tuition.

# Financial value of an MBA

## Ex. (continued)

There are 42 cash flows in this problem, so calculating the NPV by hand will take all day...

We can solve this problem easily with a **financial calculator**.

Assuming a discount rate of 8%, the **NPV of the MBA is \$83,795**.  
With these inputs, the return on an MBA is positive and substantial!

But let's consider several scenarios...

# Financial value of an MBA

## Ex. (continued)

Such an analysis is sensitive to the inputs, about which there exists some uncertainty. Therefore, it's helpful to consider a range of cases. The following table assumes the same \$75,000 tuition and \$50,000 base salary, and shows the net present value for different discount rates and post-MBA salaries:

Discount rate	<u>Post-MBA Salary</u>				
	\$60,000	\$70,000	\$80,000	\$100,000	\$150,000
3%	(\$21,305)	\$196,574	\$414,453	\$850,211	\$1,939,605
4%	(\$52,767)	\$130,229	\$313,224	\$679,215	\$1,594,191
5%	(\$76,788)	\$78,850	\$234,488	\$545,764	\$1,323,953
6%	(\$95,263)	\$38,649	\$172,560	\$440,383	\$1,109,941
7%	(\$109,558)	\$6,886	\$123,331	\$356,219	\$938,441
8%	(\$120,674)	(\$18,439)	\$83,795	\$288,264	\$799,435
9%	(\$129,346)	(\$38,804)	\$51,739	\$232,824	\$685,537
10%	(\$136,124)	(\$55,305)	\$25,514	\$187,151	\$591,244

# Returns on Social Security



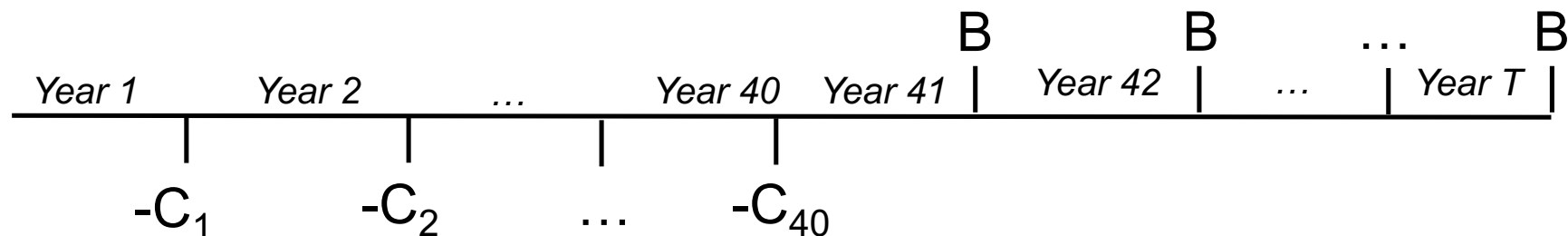
# Returns on Social Security

Social Security is not an investment, per se, but the program does exhibit characteristics similar to an investment: an individual pays into the system for a number of years until retirement, at which point the individual begins collecting payments from the Social Security Administration.

The IRR methodology may be used to calculate the implied “return” on a worker’s Social Security contributions.

# Returns on Social Security

Assume a Social Security participant works for 40 years between the ages 25-65 and makes annual contributions equal to 12.4% of annual income. The beneficiary then collects Social Security benefits throughout retirement. Assuming contributions of  $C_t$  at time  $t$  and fixed benefit payments of  $B$ , the participant's cash flow diagram will be:



Assume the beneficiary lives through  $T$  years. The return on Social Security,  $R$ , must satisfy the following formula:

$$0 = \sum_{t=1}^{40} \frac{-C_t}{(1+R)^t} + \sum_{t=41}^T \frac{B}{(1+R)^t}$$



# Returns on Social Security

**Ex 1.** Assume beneficiary makes an average salary of \$50,000 per year, which is close to the country's median salary. In this case, the annual contribution, at the contribution rate of 12.4%, is  $0.124 \times \$50,000 = \$6,200$ . At a salary of \$50,000 a year, the beneficiary is entitled to an estimated annual benefit of approximately \$22,000 (the details of benefits computation are rather complex, but simplified in this example). In this case the total benefits and returns for different longevities are:

<u>Age of Death</u>	<u>Return</u>
75	-0.48%
85	1.87%
95	2.77%
105	3.22%

(The probabilities of living to age 75, 85, 95, and 105 given an age of 25 are about 70%, 40%, 8% and 0.2%, respectively.)

# Bond prices and interest rates

If interest rates rise, what will typically happen to bonds prices?

- a) They will rise
- b) They will fall
- c) They will stay the same
- d) There is not relationship between bond prices and interest rates
- e) Do not know
- f) Prefer not to say

Source: 2015 National Financial Capability Survey

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- c) They will stay the same
- d) There is not relationship between bond prices and interest rates
- e) Do not know
- f) Prefer not to say

**28%** answered correctly.

Source: 2015 National Financial Capability Survey

# Today we learned...

- ✓ Time value of money
- ✓ Present value (PV)
- ✓ Bond prices
- ✓ Stock prices
- ✓ How to use net present value (NPV) and internal rate of return (IRR)
  - ✓ Business applications
  - ✓ Valuing an MBA
  - ✓ Returns on Social Security

# Using a Financial Calculator

## Lecture 2.2



# Financial calculators

Financial calculators can be used to solve many financial problems. In this course, we will use financial calculators to solve two types of problems:

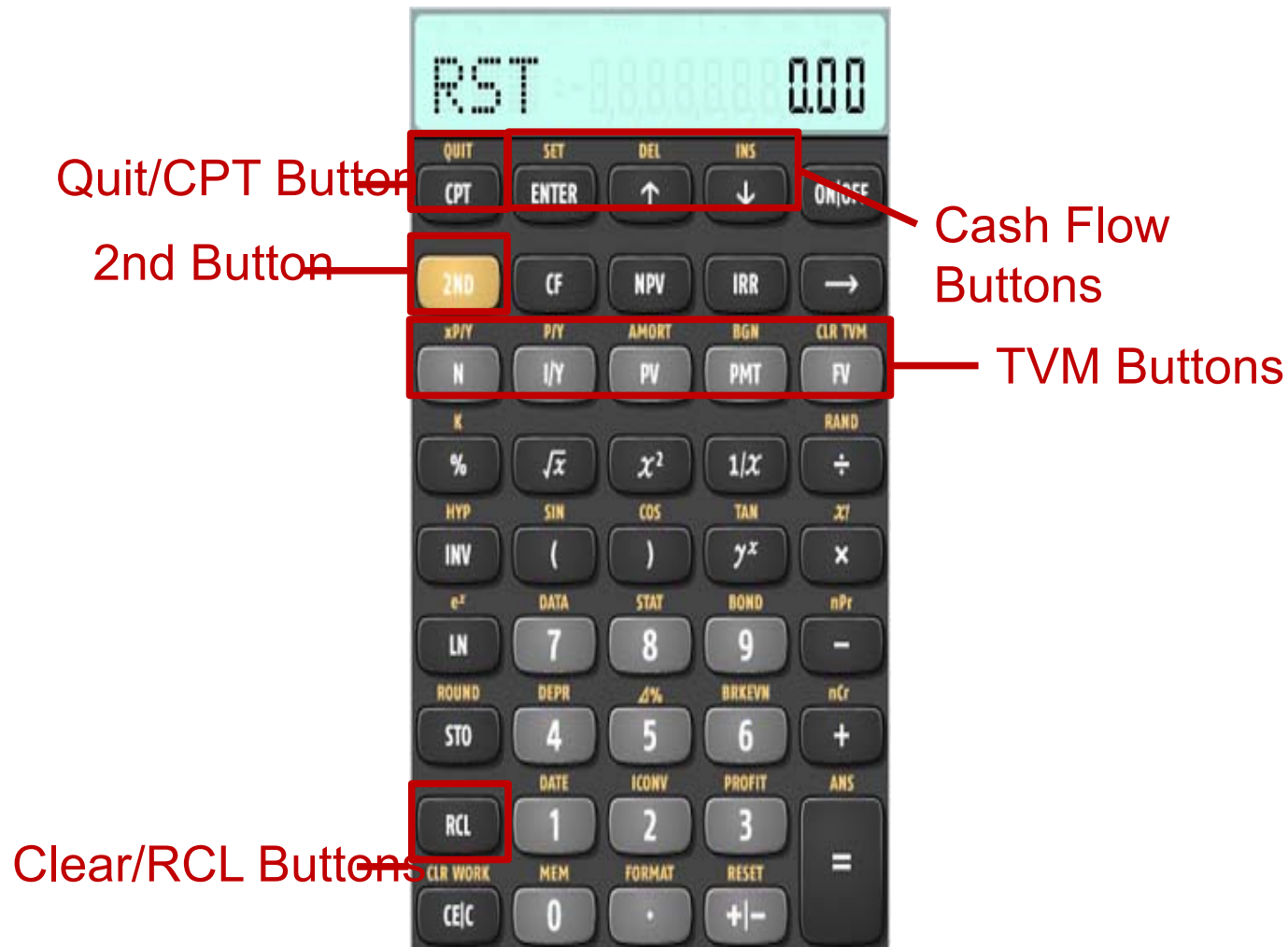
- Time Value of Money (TVM) problems.
- Cash Flow Worksheet problems.

This section will describe these types of problems and show how a financial calculator can be used to quickly solve the types of problems introduced in this lecture.

Please familiarize yourself with your financial calculator before the next lecture and make sure you're able to use your calculator to solve the problems in this section. That way, we can spend less energy on the algebra, and focus on the finance!

# Financial calculators

Today on your *TI BA II Plus*, we will use the following buttons:



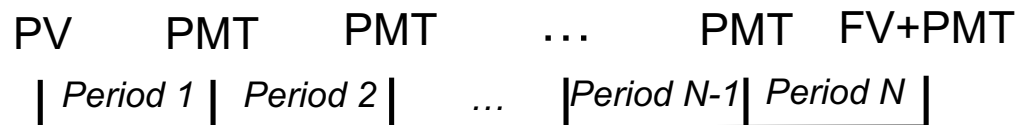
# **Time Value of Money Problems**





# TVM problems

**Time Value of Money (TVM)** problems involve cash flows that fit the following structure:



*Interest Rate = I/Y, Frequency = P/Y*

Where a negative value indicates an outflow and:

PV = Present value, initial cash flow, or principal balance

PMT = Periodic payment

FV = Final cash flow or ending balance

I/Y= Interest rate per year

N = Number of periods

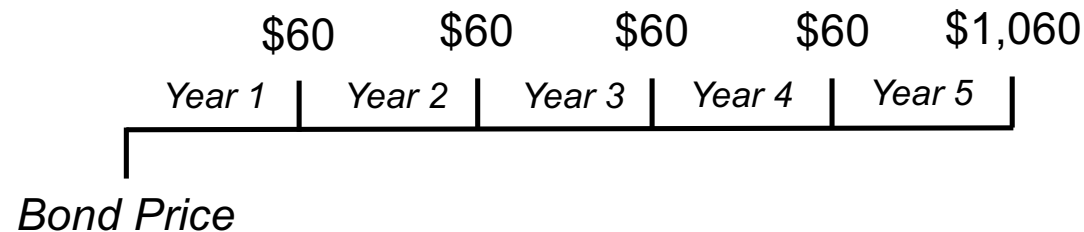
P/Y = Payments per year

TVM problems require setting the payments per year ( $P/Y$ ) and providing four of the five main inputs ( $PV$ ,  $PMT$ ,  $FV$ ,  $I/Y$ , and  $N$ ). The calculator will then compute the remaining value.

# TVM: Pricing a bond

Earlier, we calculated the price of a bond as the present value of its cash flows.

**Ex.** We looked at \$1,000 worth of a five-year corporate bond that pays annual coupons of 6%, or \$60. The cash flows for this bond are:



We found that the price of this bond is \$1,043 when the discount rate is 5%.

Because these cash flows fit the Time Value of Money structure, we can easily calculate this price using a financial calculator...

# TVM: Pricing a bond

## Ex. (continued)

The cash flows may be put into the TVM structure as follows:

PV	\$60	\$60	\$60	\$60	\$1,000 + \$60
Year 1	Year 2	Year 3	Year 4	Year 5	

*Interest Rate = 5%, Frequency = 1*

And the present value, which is the price of the bond, can be found accordingly:

<u>Time Value of Money</u>	
<b>P/Y</b>	1
<b>PMT</b>	\$60
<b>N</b>	5
<b>FV</b>	\$1,000
<b>I/Y</b>	5%
<b>PV=</b>	-\$1,043.29

This \$1,043 is the price we found earlier. Note that it is negative because a bond-buyer must *pay* \$1,043 to *receive* the \$60 and \$1,000 payments.

# TVM: Pricing a bond

With your *TI BA II Plus* financial calculator, the keystrokes are:

**Step (1/3):** Set the payments per year to 1:

To	Press	Display
Set payments per year to 1.	[2nd], [P/Y], 1, [Enter]	P/Y = 1.00
Return to standard-calculator mode	[2nd], [Quit]	0.00

**Step (2/3):** Input your four known TVM inputs:

To	Press	Display
Enter periodic payment	60, [PMT]	PMT = 60.00
Enter number of payments	5, [N]	N = 5.00
Enter final payment	1000, [FV]	FV = 1,000
Enter annual interest rate	5, [I/Y]	I/Y = 5.00

**Step (3/3):** Solve for the remaining unknown TVM value (PV):

To	Press	Display
Compute the present value	[CPT][PV]	PV = -1,043.29

# TVM: Pricing a bond

Earlier, we also claimed that the price of the bond falls to \$959 if the interest rate increases from 5% to 7%. We can easily calculate that with a *TI BA II Plus*.

**Step (1/3):** Verify that the calculator stored the previous TVM inputs:

To	Press	Display
Verify that the FV is still 1,000	[RCL], [FV]	FV = 1,000.00
Verify that PMT is still 60	[RCL], [PMT]	PMT = 60.00
Verify that N is still 5	[RCL], [N]	N = 5.00

**Step (2/3):** Change the interest rate:

To	Press	Display
Enter new interest rate	7, [I/Y]	I/Y = 7.00

**Step (3/3):** Compute the new price (PV):

To	Press	Display
Compute the new price	[CPT], [PV]	PV = -959.00

# Clearing the TVM inputs

When you're finished with a set of TVM problems, it is good practice to clear the stored values from memory.

**Step (1/2):** Clear the TVM inputs:

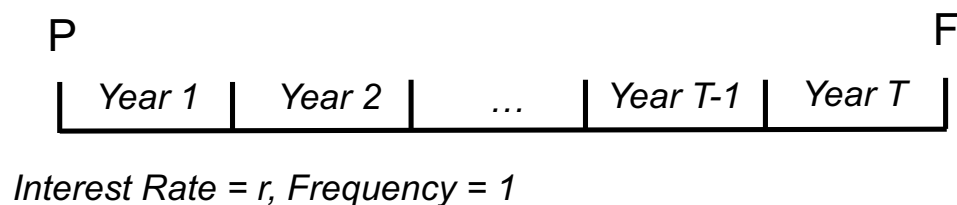
To	Press	Display
Clear TVM inputs	[2nd], [CLR TVM]	959.00

**Step (2/2):** Verify that all of the TVM inputs are now zero:

To	Press	Display
Verify that N is now zero	[RCL], [N]	N = 0.00
Verify that I/Y is now zero	[RCL], [I/Y]	I/Y = 0.00
Verify that PV is now zero	[RCL], [PV]	PV = 0.00
Verify that PMT is now zero	[RCL], [PMT]	PMT = 0.00
Verify that FV is now zero	[RCL], [FV]	FV = 0.00

# Interest compounding and discounting

The TVM function can also be used to solve simple interest compounding and discounting functions. Interest compounding and discounting problems have the following structure:



Note that this is a TVM problem with the periodic payment set to zero.

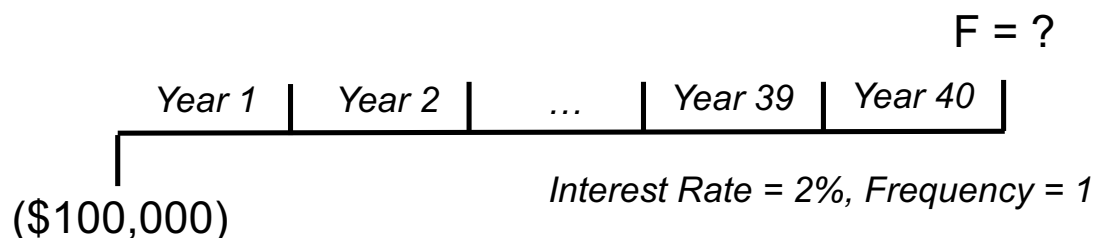
# TVM and interest compounding

**Ex.** Earlier, we saw that that \$100,000 deposited into a bank account earning 2% interest will more than double to \$220,804 in 40 years.

This can be calculated manually:

$$F = P(1 + r)^T = \$100,000 * (1.02)^{40} = \$220,804$$

Or as a TVM problem. The cash flow diagram for this problem is:



Note that the initial deposit is a cash *outflow* and must be entered as a negative in a financial calculator. **Mind your cash outflows and inflows** – if you don't enter cash flows with the correct signs, you will get odd results...



# TVM and interest compounding

## Ex. (continued)

A financial calculator can be used to find the final value (note that the final value is positive because a bank withdrawal is a cash *inflow*):

<i>Time Value of Money</i>	
<b>P/Y</b>	1
<b>PV</b>	-\$100,000
<b>N</b>	40
<b>PMT</b>	\$0
<b>I/Y</b>	2%
<hr/>	
<b>FV=</b>	\$220,804

To	Press	Display
Enter deposit amount	100000, [+ -], [PV]	PV = 100,000.00
Enter number of periods	40, [N]	N = 40.00
Set payment to zero	0, [PMT]	PMT = 0.00
Enter annual interest rate	2, [I/Y]	I/Y = 2.00
Compute final balance	[CPT], [FV]	FV = 220,803.97

Note: The payments per year (P/Y) should be set to 1 from the last problem. If not, you must set it.

# TVM and discounting

**Ex.** Earlier, we saw that the present value of \$110,000 to be received 10 years from now is \$42,410 when the interest rate is 10%.

This can be calculated manually:

$$PV = \frac{F}{(1 + r)^T} = \frac{\$110,000}{(1.10)^{10}} = \$42,410$$

Or as a TVM problem. The cash flow diagram for this problem is:



*Interest Rate = 10%, Frequency = 1*

This can be solved with a financial calculator. What do you think the sign on P will be? Why?

# TVM and discounting

**Ex. (continued)** With a financial calculator:

<u>Time Value of Money</u>	
<b>P/Y</b>	1
<b>FV</b>	\$110,000
<b>N</b>	10
<b>PMT</b>	\$0
<b>I/Y</b>	10%
<b>PV=</b>	-\$42,410

The sign of the present value is negative. This is because you must *pay* \$42,410 today in exchange for \$110,000 in ten years. If it was positive, that would mean it would be fair if somebody gave you \$42,410 today *and* \$110,000 in ten years!

To	Press	Display
Enter final value	110000, [FV]	FV = 110,000.00
Enter number of periods	10, [N]	N = 10.00
Set payment to zero	0, [PMT]	PMT = 0.00
Enter annual interest rate	10, [I/Y]	I/Y = 10.00
Compute present value	[CPT], [PV]	PV = -42,409.76

Note: The payments per year (P/Y) should be set to 1 from the earlier. If not, you must set it.

# Cash Flow Worksheet Problems



# Cash Flow Worksheet problems

**Cash Flow Worksheet** problems are the second type of problem we will use our financial calculators to solve. In these problems, we input an arbitrary series of cash flows:

$$\begin{array}{ccccccc} CF_0 & C_1 & C_2 & \dots & C_{N-1} & C_N \\ \hline & \text{Period 1} & \text{Period 2} & \dots & \text{Period N-1} & \text{Period N} \end{array}$$

And, given an interest rate, we can compute the **NPV**.

Or, we can just compute the **IRR**.

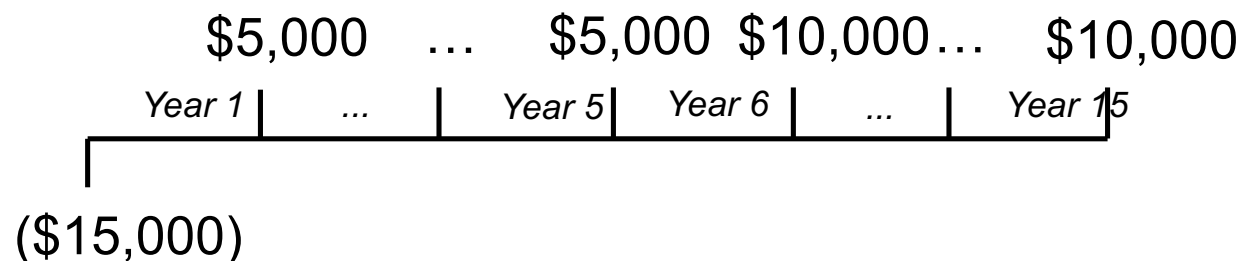
# Cash Flow Worksheet: NPV and IRR

**Ex.** Consider a business project that requires an initial investment of \$15,000. It will pay \$5,000 a year for the first 5 years, then \$10,000 per year for the next 10 years.

Calculate the NPV of this project using a discount rate of 20%.

Then calculate the IRR.

**Ans.** The cash flows of the project are:



These cash flows may be input into a financial calculator and the NPV and IRR can be computed...

# Cash Flow Worksheet: NPV and IRR

## Ans. (continued)

The initial cash flow is *negative* \$15,000. The first periodic cash flow is \$5,000 and occurs for five years. The second periodic cash flow is \$10,000 and occurs for ten years. Inputting these into a financial calculator we can solve for the NPV and IRR:

<u>Cash Flow Worksheet</u>	
<b>CF<sub>0</sub></b>	-\$15,000
<b>C01</b>	\$5,000
<b>F01</b>	5
<b>C02</b>	\$10,000
<b>F02</b>	10
<b>I</b>	20%
<hr/>	
<b>NPV=</b>	\$16,801.67
<hr/>	
<b>IRR=</b>	39.24%

# Cash Flow Worksheet: NPV and IRR

With your *TI BA II Plus* financial calculator, first enter the cash flows into the Cash Flow Worksheet. The keystrokes are:

**Step (1/4):** Open the Cash Flow Worksheet and enter the cash flows:

To	Press	Display
Open cash flow worksheet	[CF]	CF0 = 0.00
Enter today's cash flow (as a cost)	15000, [+ -], [ENTER]	CF0 = -15,000.00
Move to first cash flow	[DOWN ARROW]	C01 = 0.00
Enter first cash flow	5000, [ENTER]	C01 = 5,000.00
Move to first cash flow frequency	[DOWN ARROW]	F01 = 0.00
Enter first cash flow frequency	5, [ENTER]	F01 = 5.00
Move to second cash flow	[DOWN ARROW]	C02 = 0.00
Enter second cash flow	10000, [ENTER]	C02 = 10,000.00
Move to second cash flow frequency	[DOWN ARROW]	F02 = 0.00
Enter second cash flow frequency	10, [ENTER]	F02 = 10.00



# Cash Flow Worksheet: NPV and IRR

It is also good practice to verify that you've entered the cash flows correctly. You can scroll through the worksheet to see the cash flows you've entered.

**Step (2/4):** Scroll through the Cash Flow Worksheet to check your inputs:

To	Press	Display
Move to third cash flow	[DOWN ARROW]	C03 = 0.00
Move to third cash flow frequency	[DOWN ARROW]	F03 = 0.00
Move to today's cash flow	[DOWN ARROW]	CF0 = -15,000.00
Move to first cash flow	[DOWN ARROW]	C01 = 5,000.00
Move to first cash flow frequency	[DOWN ARROW]	F01 = 5.00
Move to second cash flow	[DOWN ARROW]	C02 = 10,000.00
Move to second cash flow frequency	[DOWN ARROW]	F02 = 10.00

Make sure all inputs are correct in the display. If they are not, re-enter the any incorrect inputs. Note that you can also move through the worksheet using the [UP ARROW] button.

# Cash Flow Worksheet: NPV and IRR

After your cash flows are stored, compute the NPV and IRR.

**Step (2/4):** Open the NPV Worksheet and compute the NPV:

To	Press	Display
Open NPV worksheet	[NPV]	I = 0.00
Enter interest rate	20, [ENTER]	I = 20.00
Move to NPV	[DOWN ARROW]	NPV = 0.00
Compute NPV	[CPT]	NPV = 16,801.67

**Step (3/4):** Open the IRR Worksheet and compute the IRR:

To	Press	Display
Open IRR worksheet	[IRR]	IRR = 0.00
Compute IRR	[CPT]	IRR = 39.24

# Clearing the Cash Flow Worksheet

When you're finished with a Cash Flow Worksheet problem, it is good practice to clear the Cash Flow Worksheet.

## **Step (1/2):** Clear the Cash Flow Worksheet:

To	Press	Display
Open cash flow worksheet	[CF]	CFO = -15,000
Clear cash flow worksheet	[2nd], [CLR WORK]	CFO = 0.00

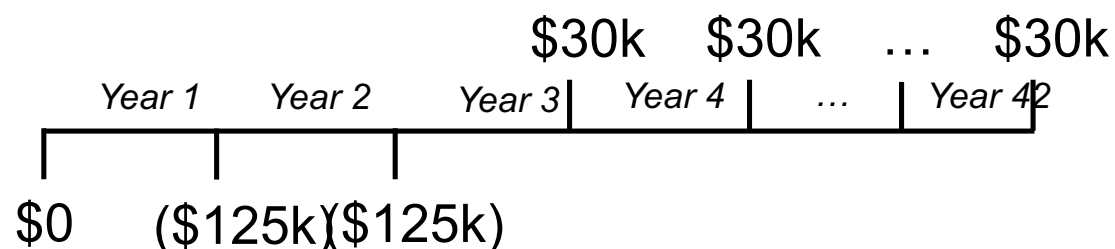
## **Step (2/2):** Exit the Cash Flow Worksheet:

To	Press	Display
Return to standard-calculator mode	[2nd], [Quit]	0.00

# NPV of an MBA

Earlier, we found the NPV an MBA. This required too many cash flows to compute by hand, so let's do it using a financial calculator.

The cash flows were:



And we discounted the cash flows using an interest rate of 8%.

Can you see what the inputs should be for a financial calculator?

# NPV of an MBA

To compute the NPV of an MBA, you should input the following into your financial calculator:

<u>Cash Flow Worksheet</u>	
<b>CF<sub>0</sub></b>	\$0
<b>C01</b>	-\$125,000
<b>F01</b>	2
<b>C02</b>	\$30,000
<b>F02</b>	40
<b>I</b>	8%
<hr/>	
<b>NPV=</b>	\$83,794.92

This is the same answer we claimed earlier.

Do you remember what keys to press to calculate this NPV?

# NPV of an MBA

With your *TI BA II Plus* financial calculator, the keystrokes are:

To	Press	Display
Open cash flow worksheet	[CF]	CF0 = 0.00
Enter today's cash flow	0, [ENTER]	CF0 = 0.00
Move to first cash flow	[DOWN ARROW]	C01 = 0.00
Enter first cash flow	125000, [+ -], [ENTER]	C01 = -125,000
Move to first cash flow frequency	[DOWN ARROW]	F01 = 0.00
Enter first cash flow frequency	2, [ENTER]	F01 = 2.00
Move to second cash flow	[DOWN ARROW]	C02 = 0.00
Enter second cash flow	30000, [ENTER]	C02 = 30,000
Move to second cash flow frequency	[DOWN ARROW]	F02 = 0.00
Enter second cash flow frequency	40, [ENTER]	F02 = 40.00
Open NPV worksheet	[NPV]	I = 0.00
Enter interest rate	8, [ENTER]	I = 8.00
Move to NPV	[DOWN ARROW]	NPV = 0.00
Compute NPV	[CPT]	NPV = 83,794.92