Optimal Financial Knowledge and Wealth Inequality[*]

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Abstract

We show that financial knowledge is a key determinant of wealth inequality in a stochastic lifecycle model with endogenous financial knowledge accumulation, where financial knowledge enables individuals to better allocate lifetime resources in a world of uncertainty and imperfect insurance. Moreover, because of how the U.S. social insurance system works, better-educated individuals have most to gain from investing in financial knowledge. Our parsimonious specification generates substantial wealth inequality relative to a one-asset saving model and one where returns on wealth depend on portfolio composition alone. We estimate that 30-40 percent of retirement wealth inequality is accounted for by financial knowledge.

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1 Introduction[1]

Americans are increasingly being asked to manage their own financial well-being during their working years as well as in retirement. This process has been hastened by the movement away from defined benefit (DB) pensions toward defined contribution (DC) plans; in 1980, about 40 percent of private-sector pension contributions went to DC plans, but two decades later, almost 90 percent of these contributions flowed to DC plans (mainly 401(k)s; see Poterba, Venti, and Wise 2007). At the same time, financial markets have become more complex, expanding the set of instruments that households can use to save and invest. The trend toward more individual responsibility means that people's financial decisions made early in life can have long-term consequences. For example, if young workers direct their pension contributions to equities instead of money market funds, this can result in quite different accumulation levels. Moreover, in this new environment, investments in financial knowledge can have important consequences for retirement well-being by influencing people's ability to save and invest. When the decision to invest in financial literacy alters life-cycle wealth profiles, individuals with similar initial circumstances can end up holding very different amounts of retirement wealth.[2] To the extent that this mechanism is at work, understanding it will help explain wealth inequality.

This paper argues that financial knowledge itself should be modeled as an endogenous choice variable akin to human capital investment. The mechanism we posit is that financial knowledge can enable individuals to better allocate resources over their lifetimes in a world of uncertainty and imperfect insurance. Our approach uses an explicit multiperiod theoretical model which allows us to explore two important questions: (1) What forces shape financial knowledge accumulation over the life cycle?, and (2) How much wealth inequality can be attributable to resulting differences in financial knowledge? We also evaluate which types of consumers would benefit most from investment in financial knowledge and the use of

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[1]An earlier version of this paper was circulated as an NBER working paper (Lusardi, Michaud, and Mitchell 2013).

[2]Wealth levels vary considerably across both workers and retirees; see Moore and Mitchell (2000) and Venti and Wise (2001).

sophisticated investment products. These issues have not been explored previously in a rich theoretical setting including uncertainty, and our answers shed light on the important issue of wealth disparity over the life cycle.

We build and calibrate a stochastic life cycle model featuring uncertainty in income, capital market returns, and medical expenditures; we also incorporate an endogenous knowledge accumulation process and a sophisticated saving technology. In the model, financial knowledge permits consumers to use sophisticated financial products which can help them raise the return earned on saving. Individuals who wish to transfer resources over time by saving will benefit most from financial knowledge. Moreover, because of how the U.S. social insurance system works (along with many other such systems around the world), better-educated individuals have the most to gain from investing in financial knowledge. As a result, making financial knowledge accumulation endogenous allows for an amplification of differences in accumulated retirement wealth over the life cycle.

Our contributions to the literature are several. First and foremost, our model endogenously generates wealth inequality above and beyond what traditional models of saving normally deliver. That is, a simple life cycle model of saving fails to replicate observed levels of wealth inequality, and little additional ground is gained by adding realism in the form of means-tested programs (e.g., Hubbard, Skinner, and Zeldes 1995), differences in preferences (e.g., Cagetti 2003), or heterogeneity in fixed costs of investing (e.g., Vissing-Jorgensen, 2003). Adding portfolio choice by fully informed, rational agents under background income and longevity risk does generate variation in portfolio composition (Wachter and Yogo 2010), but this still does not match wealth inequality on the order of what we see in the real world. Thus Venti and Wise (2001) show that permanent income differences and chance alone can explain only 30-40 percent of observed differences in retirement wealth, implying that other factors should be taken into account. By introducing endogenous variation in the returns that people can obtain on their savings, particularly on information-intensive assets, we can attribute another 30-40 percent of wealth inequality to financial knowledge.

Second, we explain why many consumers lack knowledge about key aspects of financial

markets, consistent with extensive research reporting that a large proportion of the population is not financially literate (Lusardi and Mitchell 2014). Third, we show that some level of financial ignorance may, in fact, be optimal. That is, we explain why consumers may rationally fail to invest in knowledge, since it is expensive to acquire and not everyone will benefit from greater financial sophistication. Finally, although consumers optimally choose financial knowledge given their constraints, imperfect financial knowledge still implies a meaningful welfare loss. That is, consumers would be willing to pay up to three percent of per-period consumption over their lifetimes in order to live in a world with perfect financial knowledge. Moreover, we find that a reform curtailing Social Security benefits would be anticipated to lead to higher financial knowledge.

The paper is organized as follows: we first briefly summarize prior studies; then we offer evidence on the life cycle paths of assets, consumers' use of financial products, and financial knowledge accumulation. Next we present our model, outline the model calibration, and report simulation results, along with several extensions and robustness analyses. The paper closes with conclusions and implications.

2   Prior Literature

Our work builds on several related literatures including research on household life cycle saving patterns.[3] We depart from conventional intertemporal models in that we allow for the endogenous choice of a saving technology with returns and costs that depend on a consumer's level of financial knowledge. In this way, we extend the portfolio choice literature (e.g., Cocco, Gomes, and Maenhout 2005) in which returns are assumed to be exogenous and consumers decide only how much they will invest in risky assets. Our work is also informed by prior studies that examine patterns of financial knowledge in the general population. For instance, Bernheim (1998) was among the first to note that many U.S. consumers display low levels of financial literacy. Mandell (2008) reported widespread knowledge gaps regarding

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[3]See, for instance, Cagetti (2003); De Nardi, French, and Jones (2011); Gourinchas and Parker (2002); Hubbard, Skinner, and Zeldes (hereafter HSZ 1995); and Scholz, Seshadri, and Khitatrakun (hereafter SSK 2006).

fundamental economic concepts among high school students. Lusardi and Mitchell's (2011a) Health and Retirement Study (HRS) modules on planning and financial literacy confirmed that many older individuals (age 50+) could not do simple computations nor did they grasp basic inflation and risk diversification concepts. Similar patterns also characterize younger adults (Lusardi and Mitchell 2014), and there is considerable heterogeneity in the population. Understanding the determinants of this heterogeneity is of paramount importance.[4]

Some previous researchers have also suggested that financial knowledge is an endogenously-determined choice variable. For example, Delavande, Rohwedder, and Willis (2008) posited that investment in financial knowledge is akin to human capital investment, but their static model could not trace life cycle wealth patterns. Jappelli and Padula (2013) discussed investments in financial knowledge, but their model did not explore implications for wealth inequality. Both papers built on the seminal work of Ben-Porath (1967) and Becker (1975), who modeled the economic decision to invest in human capital by linking education to wages. As with other human capital models, we assume that consumers have a basic level of sophistication regarding numeracy (e.g., knowledge of compound interest) so they can assess the returns from investing in knowledge. On a continuum from basic to sophisticated, financial knowledge incorporates the acquisition of particular skills as well as having information and awareness of financial products. Hence, in our framework, financial knowledge is not an innate trait; instead, it is acquired by investment throughout the life cycle. Our contribution is to model financial knowledge investment using a rich intertemporal setting with decision-making under many realistic sources of uncertainty; this approach permits us to evaluate the quantitative importance of financial knowledge and to perform informative policy experiments.

Our work also helps us understand recent empirical findings regarding financial knowledge and economically consequential outcomes. For example, our analysis is consistent with evidence of a positive empirical link between financial knowledge and wealth holdings.[5] Ad-

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[4]Low levels of financial skills are not only a problem in the United States, as indicated by the Organisation for Economic Co-operation and Development (OECD 2005). Lusardi and Mitchell (2011b, 2014) review a range of studies documenting low financial literacy levels around the world.

[5]See for instance Lusardi and Mitchell (2011a) and van Rooij, Lusardi, and Alessie (2012).

ditionally, our model helps explain why highly knowledgeable consumers may be more likely to participate in the stock market, which below is represented by the use of a sophisticated investment technology.[6]

Finally, our analysis speaks to the difficulty that standard life cycle models have when attempting to account for observed wealth inequality using only heterogeneity in education and permanent income. In view of the conventional model's shortcomings, some researchers have invoked a variety of factors including impatience in the form of hyperbolic discounting (Angeletos, Laibson, Repetto, Tobacman, and Weinberg 2001), or means-tested programs (HSZ 1995). By contrast, our approach draws on the fact that risk-adjusted expected returns from financial products can differ across income groups. For example, Yitzhaki (1987) established that higher earners enjoyed higher returns on stock market holdings. In experimental settings, Choi, Laibson, and Madrian (2010) and Hastings, Mitchell, and Chyn (2011) showed that more financially knowledgeable individuals paid lower fees for mutual funds. Since such fees can substantially reduce net returns on such investments, this implies that financial knowledge boosts investors' net returns. Using administrative data from a large financial institution combined with a survey, Clark, Lusardi, and Mitchell (2014) found that more financially knowledgeable employees could expect to earn higher risk-adjusted returns on their retirement savings.

One might think that financial knowledge would not be needed if individuals could rely on financial advisers, yet in practice, only a minority of the population does so. For instance, Bricker, Kennickell, Moore, and Sabelhaus (2012) reported that fewer than one-third of respondents in the U.S. Survey of Consumer Finances consult advisers, and we found similar results in the U.S. National Financial Capability Study (NFCS). Importantly, there are also large differences across education groups: only 11 percent of high school dropouts use professional advice, versus 45 percent of college graduates.[7] Thus the least-

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[6]This is supported by the empirical evidence in Christelis, Jappelli, and Padula (2010) and van Rooij, Lusardi, and Alessie (2011), among others.

[7]Only 30 percent of white men responding to the NFCS reported that they sought advice over the past five years from financial professionals regarding saving and investment. Additional detail is available from the authors upon request.

educated, who also have the least financial knowledge, are rather unlikely to turn to financial professionals for advice. It is also worth noting that there are impediments to obtaining good financial advice when consumers lack financial knowledge (U.S. GAO 2011). In other words, financial knowledge is plausibly a complement to, rather than a substitute for, financial advice (Collins 2012).

Financial knowledge could also have an effect on diversification, which may lead to higher risk-adjusted returns. As one example, Calvet, Campbell, and Sodini (2009) showed that better-educated Swedish households held more stock than the less-educated, though conditional on holding stocks, they achieved lower unsystematic risk on their portfolios. As another example, using Dutch data, von Gaudecker (2011) looked at the relationship between investment diversification (return loss), financial knowledge, and financial advice, and he reported that the least financially literate were unlikely to diversify their assets.

Large differences in returns can produce a considerable amount of wealth inequality: for example, a dollar invested at a 6% versus a 2% return over 50 years grows to be about seven times larger. Moreover, to simply assume substantial heterogeneity in returns across population groups does not help much in explaining wealth differentials, since that merely replaces one source of unexplained heterogeneity with another. Instead, our approach generates such heterogeneity arising from endogenous accumulation of financial knowledge.

3 Life Cycle Wealth and Financial Knowledge

3.1 The Evolution of Income and Assets by Education

The basic life cycle economic model posits that consumers seek to transfer resources from periods of their lives when they earn substantial income to periods when they earn less, given concavity of the utility function. We illustrate typical household income profiles over the life cycle in Figure 1, which plots median net (after-tax) household income by education group constructed from the Panel Study of Income Dynamics (PSID).[8] Educational attainment

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[8]These calculations use the PSID Cross-National Equivalent Files (CNEF) from 1984 to 2005 (in $2004), where after-tax household income excludes income from capital. To generate the figure, we first run median

refs to three sets of household heads: those who did not complete high school (<HS), high school graduates (HS), and those with at least some college (College+). We focus on white males throughout this paper to keep our sample as homogeneous as possible.

[Figure 1 here]

As is evident, life cycle household income for this cohort is hump-shaped. It rises at a faster rate for the college-educated than for the less-educated, and from around age 50 onward, income slowly decreases for all groups. Post-retirement, income falls due to the fact that Social Security and pension benefit amounts are generally less than labor earnings. In the U.S., old-age benefit replacement rates are higher for the least-educated due to the progressivity of public safety net programs, so better-educated consumers see their incomes fall relatively more after retirement. Net household income also declines somewhat for all groups in retirement, in part because of changes in household composition (e.g., loss of a spouse).

Figure 2 traces life cycle paths of median net wealth (defined as the sum of bank accounts, stocks, IRAs, mutual funds, bonds, and net real estate, minus debt) for these same individuals.[9] We drop from these calculations individuals with business assets and exclude outliers.[10] For the typical household, wealth grows steadily up to the mid-60s and then flattens or declines. Again, there are striking differences by educational attainment, with the median college-educated household having around $375,000 in wealth at age 65 (in $2004). By contrast, high school dropouts at the same age had accumulated only $125,000, with most of that in the form of housing wealth.

[Figure 2 here]

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regressions with age and cohort effects, and then we predict incomes for the 1935-1945 cohorts. Age dummies are smoothed with a lowess filter.

[9]In what follows, we refer interchangeably to net wealth, net assets, net worth, and household wealth. As the PSID does not have data on 401(k) plan balances, these are not included in wealth measures prior to retirement; nevertheless income received from 401(k) plans is included in our income measure. Furthermore, if the balance from a 401(k) plan is cashed out at retirement and rolled over to some other form of asset, it is included in our wealth measure.

[10]Including those with business assets skews the interpretation of saving motives compared to the general population, because of the large amount of wealth held in these ventures as well as the volatility of business owners' incomes (see Hurst, Kennickell, Lusardi, and Torralba 2010).

In the simplest version of the life cycle saving model, individuals will optimally consume only a portion of their lifetime incomes each period, borrowing in some periods and saving in others. A key prediction of this framework is that the life cycle path of assets normalized by lifetime income should be the same across groups. Therefore, as noted by HSZ (1995), higher earners would be predicted to have wealth profiles that simply scale-up lower earners' paths. Yet the data reveal non-proportional wealth-to-income profiles, implying that the simple life cycle model cannot explain observed wealth heterogeneity. This is consistent with evidence of large differences in saving rates across education groups in the Consumer Expenditure Survey (CEX) and the PSID (Dynan, Skinner, and Zeldes 2004).

More sophisticated models allow for a precautionary saving motive which comes into play when income is uncertain, insurance markets are imperfect, and borrowing is difficult. In these circumstances, some individuals anticipate that they will have a very high marginal utility of consumption when future income is low. Given a concave utility function exhibiting prudence, such a consumer would save more in anticipation of this possibility.[11] While precautionary saving can explain some of the heterogeneity observed in the data, it still falls short of explaining wealth differences among those facing similar uncertain income profiles. Yet another explanation for why the less-educated fail to save was offered by HSZ (1995), who noted that the U.S. social insurance system protects families with limited resources against bad states of the world. That is, means-tested and redistributive transfer programs such as Social Security, Medicaid, and Supplemental Security Income provide an explicit consumption floor in the event that households fall into poverty. In turn, the existence of such a consumption floor dampens consumers' precautionary saving motives, particularly when people are rather likely to become eligible for such benefits. Though this does help explain why the less-educated save little, it cannot explain wealth inequality in the upper half of the income distribution where the consumption floor is less likely to be binding.

Other authors appeal to differences in preferences to explain observed wealth inequality patterns. For example, Cagetti (2003) found that less-educated consumers have a high rate

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[11]See Deaton (1992).

of time preference and lower rate of risk aversion, compared to more-educated consumers. When using constant relative risk aversion preferences, lower risk aversion implies lower prudence. This leads to small precautionary saving for the less-educated and younger consumers, while college-educated consumers were more patient and more prudent. Differences in household composition over the life cycle can also affect consumption by directly changing discount factors or the marginal utility of consumption: inasmuch as household size is negatively correlated with education, this could also account for some portion of wealth inequality (Attanasio, Banks, Meghir, and Weber 1999; SSK 2006). Another potential channel generating wealth inequality might be differences in anticipated mortality patterns. It is well-known that the more educated live longer, which might also account for a portion of the observed divergence in wealth accumulation (and decumulation) across groups (De Nardi, French, and Jones 2011). Our analysis below incorporates many of these factors to assess how their impacts compare to those of endogenous financial knowledge as a separate channel accounting for wealth inequality.

3.2 Differences in Sophisticated Financial Products and Returns by Education

In view of the income paths illustrated above, it should be apparent that college-educated consumers would optimally do relatively more saving, compared to the less-educated. In turn, this could make the better-educated group more interested in a technology that enhanced returns on resources transferred across periods, compared to their less-educated peers. Table 1 shows the fraction of PSID respondents holding stocks, mutual funds, bonds, and/or individual retirement accounts (IRAs), by age and education. We denote these products as relatively "sophisticated," compared to having only a bank account (or no savings at all).

[Table 1 here]

Compared to high school dropouts, better-educated households are far more likely to use such sophisticated products for saving.[12] In particular, more than three-quarters of the older (age 55-65) College+ group use sophisticated products, compared to about one-quarter of the high school dropouts of the same age. We also note that the better-educated hold a larger share of their financial wealth in sophisticated products at all ages, conditional on using them: for those age 55-65, these account for almost 60 percent of the assets of the College+, but only 50 percent for the least-educated. Importantly, differences across education groups at the intensive margin (how much to invest in the products) are much smaller than differences at the extensive margin (using the same products). Provided that sophisticated products yield higher returns, these differences in the composition of savings could generate substantial differences in wealth levels at retirement.

The possibility that the more educated and better-paid enjoy higher risk-adjusted returns may result from greater knowledge about financial products. For instance, some authors surmise that lack of financial knowledge can explain people's generally low levels of investment and low participation in the stock market (van Rooij, Lusardi, and Alessie 2011). Empirical information on the evolution of financial knowledge over the life-cycle may be gleaned from the NFCS.[13] The NFCS contains five questions measuring knowledge about compound interest, inflation, risk diversification, basic asset pricing, and mortgage interest payments; some of these questions have been used to measure levels of financial knowledge not only in the U.S. but also around the world (Lusardi and Mitchell 2011b, 2014). The left-hand panel of Figure 3 reports the fraction of correct answers by education and age in this survey, and four aspects are worth highlighting. First, financial knowledge differs considerably across education groups. More than 40 percent of the middle-aged College+ respondents answer all five questions correctly, but only 10 percent of high school dropouts do. Second, financial knowledge increases over the life cycle, consistent with the idea that consumers accumulate knowledge as they age. Third, observed differences in financial knowledge widen with age: the better-educated tend to accumulate more financial knowledge just prior to retirement. A

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[12]See also Curcuru, Heaton, Lucas, and Moore (2005) and Campbell (2006).

[13]Here we refer to the 2012 survey; see http://www.usfinancialcapability.org

regression of financial knowledge on age, education, and age/education interactions reveals that the interactions are statistically significant (the partial F-test statistic for the interactions is 5.6 with a p-value<0.001), providing evidence that differences across education groups grow differentially with age. Fourth, there is some evidence that financial knowledge eventually decreases past middle age (consistent with Lusardi and Mitchell 2011a). The right-hand panel of Figure 3 displays the percentage of respondents by educational group who say they use financial advisers. In a regression framework, we confirm that few people use advisers, particularly among the least-educated. This indicates that advice is probably a complement to, rather than a substitute for, financial knowledge acquisition.

[Figure 3 Here]

Equity investments provide higher expected returns than do bonds, so one might anticipate that better-educated households would earn higher returns than their lower-educated counterparts. Yet such differences in financial holdings are insufficient to generate observed wealth inequality patterns; for instance, Venti and Wise (2001) reported that including controls for stock ownership contributed little to explaining the dispersion of wealth across households. That is, adding "investment choices" as controls (in addition to lifetime earnings) reduces the unconditional standard deviation of wealth at retirement by only 8 percent, at the margin. Accordingly, if the differential take-up of sophisticated products is to account for a larger share of wealth inequality, researchers must allow for the possibility that returns are persistently heterogeneous across households in a predictable way. In fact, there is evidence that financial knowledge is positively associated with investment returns. Lab experiments by Choi, Laibson, and Madrian (2010) found that people who deemed themselves more financially knowledgeable did elect lower fee investment options than their counterparts, a finding confirmed in other studies examining fees and retirement saving (Hastings, Mitchell, and Chyn 2011). Jappelli and Padula (2013) cited additional evidence on the relationship between interest rates and proxies for financial literacy. Most importantly, they showed that those who correctly answered more financial literacy questions got higher returns on their

savings, evidence corroborated by self-reported information on what interest rates people thought they could earn. Using administrative data from a large financial institution, Clark, Lusardi, and Mitchell (2014) found that more financially knowledgeable employees earned higher expected risk-adjusted returns on their 401(k) accounts compared to those with little financial knowledge.

3.3 Financial Knowledge and Wealth Accumulation

To illustrate how financial knowledge can alter the invariance of wealth to income in a standard life cycle model, we first offer a two-period example. Subsequently, we extend the model to incorporate a richer framework. Accordingly, we assume that the consumer receives labor income y only in the first period. Denoting wealth in period 2 as a, we seek to understand wealth accumulation in period 2 as a function of lifetime income. The consumer can choose how much to consume, c, in the first period, and how much to invest in raising R, the return factor on saving, s. For simplicity, we assume there is a one-to-one mapping between knowledge and R. Thus, $a = Rs$ and $c = y - \pi R - a/R$ where π is the monetary cost of raising R by one unit. Assuming the consumer has a discount factor β, he maximizes:

$$\max_{a,R} u(y - \pi R - a/R) + \beta u(a)$$

Assuming log utility, $u(c) = \log c$, and fixed R and thus no investment in knowledge, the solution for wealth is proportional to income, and therefore normalized optimal wealth is a constant ($\frac{a^*}{y} = \frac{R\beta}{1+\beta}$). But when we maximize jointly over a and R, we obtain the following solution for optimal wealth as a ratio of income:

$$\frac{a^*}{y} = \frac{y}{(2 + \frac{1}{\beta})^2 \pi}.$$

Optimal wealth as a ratio of income increases with income, a finding that breaks the in-

variance property.[14] In a model with endogenous knowledge, there is a complementarity between an agent's need to save and his willingness to invest to raise R. For high values of y, the reward to investing in R rises because saving needs are relatively important. In this two-period model, where lifetime income and the income trajectory are the same, it is not higher income *per se* that raises the incentive to invest in financial knowledge, but rather the need to smooth marginal utility between periods. The need is greater when there is a larger gap between first- and second-period consumption. Accordingly, heterogeneity in retiree benefit replacement rates can affect the incentives to invest in financial knowledge; in turn, this can lead to additional differences in wealth accumulation. The same can be said of differences in demographic factors (e.g., family size) that shift the marginal utility of consumption over the life cycle, as well as differences in expected mortality. As indicated above, the slope depends in part on the discount factor and the cost of investing in knowledge. The relationship is stronger for lower values of the cost of investing in knowledge and for higher β.

A richer setting with uncertainty and borrowing constraints offers additional motivations to save. If consumers are liquidity-constrained, they may be unwilling to invest in financial knowledge. Faced with uncertainty, the consumer might also wish to save more and invest more in financial knowledge for precautionary reasons. Furthermore, the sensitivity of saving to the interest rate can be smaller than in the certainty case (Cagetti 2003), which may also affect incentives to invest in knowledge. Accordingly, we next turn to a richer model of saving to investigate the effects of financial knowledge on wealth and wealth inequality.

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[14]Assuming power utility, the FOC ($u(c) = \frac{c^{\{1-\sigma\}}}{1-\sigma}$) for wealth is

$$a^{1-\frac{1}{2\sigma}}(y - 2\sqrt{\pi a}) = \left(\frac{\sqrt{\pi}}{\beta}\right)^{1/\sigma}$$

While the right-hand side of the equation is constant with wealth, the left-hand side is not: the left-hand side is decreasing in a for reasonable values of σ and π. A rise in income increases the left-hand side for a given wealth level. If the wealth ratio is to increase to equal the right-hand side, wealth must rise by more than income.

4 The Model

We extend the two-period example above in several directions to allow cross-sectional varia-
tion in both financial knowledge and wealth levels. First, we introduce uncertainty regarding
asset returns, household income, and out-of-pocket medical expenditures. The consumer is
assumed to choose his consumption stream by maximizing expected discounted utility, where
utility flows are discounted by β. Second, the individual also faces stochastic mortality risk,
and decisions are made from time $t = 0$ (age 25) to age T (or as long as the consumer
is still alive; $T = 100$). Third, and adding to the heterogeneity created by the stochastic
components, we also examine three different education groups (<HS, HS, and College+).
Across these groups, we allow for heterogeneity in income, mortality, demographics, and out-
of-pocket medical expenditure levels. Importantly, to highlight how investment in financial
knowledge affects outcomes, we do not allow for differences in preferences, and we assume
consumers start their life cycles with no financial knowledge. In extensions presented below,
we consider the role of alternative preferences, including allowing for taste heterogeneity.

The utility function is assumed to be strictly concave in consumption and defined as
$n_t u(c_t/n_t)$, where n_t is an equivalence scale capturing (known) differences in consumption
patterns across demographic groups (SSK 2006). The marginal utility of consumption is
$u'(c_t/n_t)$ and thus rises with n_t. Since the path of n_t is hump-shaped over the life cycle,
this contributes to generate a hump-shaped consumption profile with age (Attanasio, Banks,
Meghir, and Weber 1999).

The consumer may elect to invest his resources in two different investment technologies.
The first is a basic technology (for example, a checking account) which yields a certain (low)
return $\bar{r}$ ($\bar{R} = 1 + \bar{r}$). This represents the expected return to consumers without any financial
know-how. The second is a more sophisticated technology which enables the consumer to
receive a higher expected return, which increases in financial knowledge f but comes at a
cost. Specifically, the consumer must pay a direct cost (fee) to use the technology, c_d, and
he must also invest time and money in acquiring the knowledge to generate a sufficiently

high excess return.

Obtaining knowledge in the form of investment i_t thus has a cost of $\pi_i(i_t)$; we assume that this cost function is convex, reflecting decreasing returns in the production of knowledge.[15] Relatively little is known about how this cost might vary across individuals; for instance, it could either rise or fall depending on the level of education. Clearly the opportunity cost of time is higher for higher earners, but education might be a complement in the production of knowledge, making it easier for the better-educated to learn. We remain agnostic about whether the average cost of investing in additional knowledge is higher or lower for more educated households; rather, we assume initially that all households face the same cost function.

The rate of return on the sophisticated technology is stochastic, with an expected return that depends on the agent's level of financial knowledge at the end of t, $\widetilde{R}(f_{t+1})$. Thus the stochastic return function is log-normally distributed with $\log \widetilde{R}(f_{t+1}) = \bar{r} + r(f_t) + \sigma_\varepsilon \varepsilon_t$ where σ_ε is the standard deviation of a normally distributed shock ε_t. The function $r(f_t)$ is increasing in f_t and it can be interpreted as an excess return function. Since the variance is assumed fixed, this also implies that agents with higher financial knowledge obtain a higher Sharpe ratio (higher risk-adjusted return) on their investments.[16] We denote by κ_t the fraction of wealth that the consumer invests in the sophisticated technology in period t.

We posit that financial knowledge evolves according to the following equation:

$$f_{t+1} = (1 - \delta)f_t + i_t$$

where δ is a depreciation rate and i_t is gross investment. Depreciation exists both because consumer financial knowledge may decay and because some knowledge may become obsolete as new financial products are developed.[17]

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[15]A convex cost function has the advantage of avoiding bang-bang solutions where consumers invest massively in one period; hence it encourages the smoothing of investment over time (see Delavande, Rohwedder, and Willis 2008).

[16]Below we also consider the possibility that financial knowledge allows a reduction in risk through diversification.

[17]An extension below also examines learning-by-doing as an alternative form of investment in knowledge.

The consumer is also eligible for a government transfer tr_t which guarantees a minimum consumption floor of c_{min} (as in HSZ 1995). This consumption floor may lower the expected variance of future consumption, which diminishes the precautionary motive for saving. Transfers are defined as $tr_t = \max(c_{min} - x_t, 0)$ where cash on hand is:

$$x_t = a_t + y_t - oop_t$$

where y_t is net household income and oop_t represents out-of-pocket medical expenditures. Both variables are stochastic over and above a deterministic trend. The sophisticated technology cannot be purchased if $x_t - c_d < c_{min}$ (that is, the government will not pay for costs of obtaining the technology). End-of-period assets are given by:

$$a_{t+1} = \widetilde{R}_\kappa(f_{t+1})(x_t + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0))$$

where $\widetilde{R}_\kappa(f_{t+1}) = (1 - \kappa_t)\overline{R} + \kappa_t \widetilde{R}(f_t)$. We impose a borrowing constraint on the model such that assets a_{t+1} must be non-negative.

As in many papers in this literature, we posit that during the worklife, the individual's net income equation (in logs) is given by a deterministic component which depends on education, age, and an AR(1) stochastic process:

$$\log y_{e,t} = g_{y,e}(t) + \mu_{y,t} + \nu_{y,t}$$

$$\mu_{y,t} = \rho_{y,e}\mu_{y,t-1} + \varepsilon_{y,t}$$

$$\varepsilon_{y,t} \sim N(0, \sigma_{y,\varepsilon}^2), \ \nu_{y,t} \sim N(0, \sigma_{y,v}^2)$$

Here e represents the education group, and $g_{y,e}(t)$ is an age polynomial (quadratic). The error term $\eta_{y,t}$ is the sum of a persistent component $\mu_{y,t}$ and an idiosyncratic component $\nu_{y,t}$. Retirement is exogenous at age 65. After retirement, the individual receives retirement benefits which are a function of pre-retirement income.

A similar stochastic AR(1) process is assumed for out-of-pocket medical expenditures.

Out-of-pocket expenditures (in logs) follow the process described below:

$$\log oop_{e,t} = g_{o,e}(t) + \mu_{o,t} + \nu_{o,t}$$

$$\mu_{o,t} = \rho_{o,e}\mu_{o,t-1} + \varepsilon_{o,t}$$

$$\varepsilon_{o,t} \sim N(0, \sigma_{o,\varepsilon}^2), \ \nu_{o,t} \sim N(0, \sigma_{o,v}^2).$$

Because these expenditures are generally low prior to retirement (and to save on computation time), we allow only for medical expenditure risk after retirement (as in HSZ 1995). Again, the error term $\eta_{o,t}$ is the sum of a persistent component $\mu_{o,t}$ and an idiosyncratic component $\nu_{o,t}$. Finally, we allow for mortality risk at all ages, denoting $p_{e,t}$ as the one-year survival probability. Mortality risk is allowed to differ across education groups.

The state-space in period t is defined as $s_t = (\eta_{y,t}, \eta_{o,t}, e, f_t, a_t)$. The consumer's decisions are given by (c_t, i_t, κ_t). Hence there are three continuous control variables (consumption, investment, and the share of investment in the technology). There are five state variables. We represent the problem as a series of Bellman equations such that, at each age, the value function has the following form where :[18]

$$V_d(s_t) = \max_{c_t, i_t, \kappa_t} n_{e,t} u(c_t/n_{e,t}) + \beta p_{e,t} \int_{\varepsilon} \int_{\eta_y} \int_{\eta_o} V(s_{t+1}) dF_e(\eta_o) dF_e(\eta_y) dF(\varepsilon)$$

$$a_{t+1} = \widetilde{R}_\kappa(f_{t+1})(a_t + y_{e,t} + oop_{e,t} + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0))$$

$$f_{t+1} = (1 - \delta) f_t + i_t$$

$$\widetilde{R}_\kappa(f_{t+1}) = (1 - \kappa_t)\overline{R} + \kappa_t \widetilde{R}(f_t).$$

where $a_{t+1} \geq 0$. We index variables by e where education differences are assumed to be present.[19]

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[18]This formulation abstracts from bequest motives. While an extension to include bequests could be interesting, the evidence suggests that this would have a minimal effect on wealth decumulation among the elderly (De Nardi, French, and Jones 2011). Moreover, incorporating bequests would increase wealth inequality without changing the qualitative nature of our results.

[19]There are four sources of risk over which the value function is integrated: mortality, rate of return,

The model is solved by backward recursion after discretizing the continuous state variables. At each point in the state-space, we use a simplex method (Nelder-Mead) to search for the optimal solution of consumption, financial knowledge investment, and investment in the sophisticated technology. We also evaluate utility at corner solutions because of kinks in the objective function. We solve for optimal decisions for a grid of 40 net asset points and 25 financial knowledge points. Bi-linear interpolation is used to find the value function when net assets or the financial knowledge stock at $t+1$ fall off the grid; the value function behaves smoothly and is concave except at low levels of net assets where liquidity constraints and the consumption floor bind. Accordingly, the grid for assets in the state-space is defined as equally spaced points on $a^{0.3}$, which leads to more points at lower levels of net assets. We use the method proposed by Tauchen (1986) to discretize the processes for income and out-of-pocket median expenditures (with seven points each). Finally, we use seven points for rate of return shocks. The resulting decision rules are smooth and well-behaved.

5   Calibration

Our goals are to show how endogenous financial knowledge affects wealth holding, and to understand the determinants of financial knowledge accumulation patterns. Since we lack information on returns by education group over the life cycle, we do not estimate all relevant parameters of the model. Rather, we proceed with a calibration using plausible values from the literature for preferences and constraints for our base case. Additionally, we provide results from extensive sensitivity analyses in Section 6.

To implement the model in the base case, we assume that $u(c_t/n_t)$ has a CRRA form with relative risk aversion σ. The value of 3 for this parameter used by HSZ (1995) is reasonable in their context, since their main mechanism for creating dispersion in saving patterns is the differential impact of the precautionary saving motive due to a consumption floor. Accordingly, their precautionary saving motive governed by the coefficient of relative prudence, $1 + \sigma$, needed to be large. By contrast, our model has an additional channel

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out-of-pocket medical expenditures, and income. These risks are assumed to be independent.

for creating wealth dispersion, so there is no need for such a strong precautionary saving motive. We use a value of $\sigma = 1.6$ in the base case, which is close to the value estimated by Attanasio, Banks, Meghir, and Weber (1999) using consumption data. It is worth noting that the portfolio choice literature typically assumes risk aversion parameters in excess of 4 (e.g. Campbell and Viceira 2002; Cocco, Gomes, and Maenhout 2005), but we do not require such a high degree of risk aversion in our model. One reason is that agents with low financial knowledge already face low returns if they use the sophisticated technology; hence they will not adopt it. Additionally, the cost of participating in the sophisticated technology reduces the incentives to use it (Vissing-Jorgensen 2003). Both factors imply that we can fit market participation patterns relatively well using the sophisticated technology proposed here, without resorting to high values of risk aversion.

Following SSK (2006), we define an equivalence scale that accounts for consumption differences in household size by education group and changes in family composition over the life cycle. Let $z(j, k) = (j + 0.7k)^{0.75}$ where j is the number of adults in the household and k is the number of children (under 18 years old). We then define $n_{e,t} = z(j_{e,t}, k_{e,t})/z(2, 1)$ where $j_{e,t}$ and $k_{e,t}$ are the average number of adults and children in the household by age and education group, and $z(2, 1)$ be the equivalence scale for a household with two adults and one child. We use PSID data to estimate the time series of average equivalence scales by education group. The age profile of those scales is hump-shaped and more amplified for less-educated households.[20] For the base case, we use a discount factor of 0.96 (as in SSK, 2006, and Campbell and Viceira 2002).[21] The annual minimum consumption floor is set at $10,000 per couple with one child.[22]

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[20]In the PSID, we compute the average number of adults and children (under 18 years old) per household, according to the head's education and age. We then implement the equivalence scale according to the formula in the text.

[21]This is also close to the value of De Nardi, French, and Jones (2011) who estimate it to be 0.97, and Cagetti (2003) who estimates a value of 0.948 for high school dropouts and 0.989 for the college-educated.

[22]This value is derived from the Assistant Secretary for Planning and Evaluation (ASPE, 2008), where the maximum monthly benefit payable to a couple with one child under the Temporary Assistance for Needy Families (TANF) program was $495 (in $2006). The average monthly benefit of recipients on food stamps (for a 3-person household) was $283. Hence, prior to age 65, the sum of TANF and food stamp benefits totaled $778/month for a 3-person household or $9,336/year (omitting the lifetime TANF receipt limit). The Social Security Administration (http://www.ssa.gov/pressoffice/factsheets/colafacts2004.htm) reports that the 2004 maximum monthly federal payment for SSI was $552 for a single household and $829 for couples;

Post-retirement income is a function of pre-retirement income, estimated from fixed-effect regressions analyzed separately by education level of net household income on age and a retirement dummy. This produces replacement rates of 0.75 for high school dropouts, 0.74 for high school graduates, and 0.63 for the College+, close to those based on total retirement income in the literature (e.g., Aon Consulting 2008). These are higher than rates based only on Social Security benefits, since older households have additional sources of retirement income (e.g. spousal earnings, employer pension benefits, annuities, etc.). Following retirement, we let income decline at the rate estimated in PSID data controlling for educational groups and cohort effects; that pattern is mostly due to changes in household composition (e.g. widowhood).

The return on the safe asset is set to $\bar{r} = 2\%$ (as in Campbell and Viceira 2002). The form of the excess return function is unavailable from previous studies. The range of risk-adjusted excess portfolio returns reported, for example, by von Gaudecker (2011) is -0.017 (5th percentile) to 0.054 (95th percentile).[23] Using Euler equations, Jappelli and Padula (2013) estimated that each point of financial literacy is associated with an expected increase in the return on saving from 0.2 to 1 percent. For simplicity, we employ a linear function by setting $r_{max} = r(f_{max}) = 0.04$ and $r_{min} = r(f_{min}) = 0$ where 0.04 is chosen to match the equity premium used in the portfolio literature. Below, we choose a convex cost function for investing in financial knowledge which therefore embodies decreasing returns to producing knowledge. Accordingly, even if the production function is linear, agents will seek to smooth their investments in financial knowledge over the life cycle. We adopt this simpler form of the production function in order to show the basic mechanisms of the model. In robustness analysis reported later, we show how allowing for a concave relationship between financial knowledge and returns affects results regarding the role of financial knowledge in explaining wealth inequality. We set $\sigma_\varepsilon = 0.16$ in the simulations (Campbell and Viceira, 2002).

To compute the deterministic part of net household income, we draw on PSID data from

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including food stamps yields an annual total of $7,620 for singles and $12,180 for couples. Accordingly we use a value of $10,000/year in $2004, comparable to the $12,000 used by HSZ (1995).

[23]We thank Hans-Martin von Gaudecker for sharing these statistics from the Dutch portfolio data with us.

the Cross-National Equivalent files, pooling all available waves (1980-2005).[24] The NBER's Taxsim program is used to compute net household income. We account for cohort effects when computing income profiles, setting the cohort effect for our calibration baseline to the 1935-1945 birth group. For comparability with prior studies, we use the AR(1) error structure estimated by HSZ (1995) for net household income prior to age 65. Earnings are quite persistent: the autocorrelation coefficient is set to 0.95, constant across education groups, while the variance of the innovation decreases with education (0.033 for high school dropouts, to 0.016 for the College+). We use HRS data to compute the profile of household out-of-pocket medical expenditures allowing for cohort effects; we predict the profiles for those born between 1935 and 1945, again using the error structure estimated by HSZ (1995). Expenditures also prove to be quite persistent. The auto correlation coefficient is therefore set to 0.901, constant across education groups. The variance of the innovation ranges from 0.175 for dropouts, to 0.153 for the College+. Following the literature (HSZ, 1995; SSK, 2006), we set the variance of the transitory error component to zero in the simulations since most of it likely reflects measurement error.

Estimating the price of acquiring financial knowledge is difficult because little information is available on inputs to the production process – time and expenditures on financial services – let alone data on investments in, as opposed to the stock of, financial knowledge. According to Turner and Muir (2013), the cost of a one-hour financial advice consultation averages about $250. Veritat.com offers financial planning at $25 a month for singles and $40 for families ($35 for retirees), after an initial meeting fee of $250. Accordingly, the cost can range from $550 for singles to $730 for families. Less-expensive alternatives include financial advice software such as ESPlanner, where a one-year license costs $40 (the upgraded ESPlanner costs $149; see esplanner.com/product_catalog). Since the first units of knowledge are probably cheap to acquire and marginal cost probably rises quickly, we use the function $\pi(i_t) = 50i_t^{1.75}$. For the participation cost of the sophisticated technology (c_d), we use the median estimate of $750 (in $2004) from Vissing-Jorgensen (2003).

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[24]See https://cnef.ehe.osu.edu/.

We also require an estimate of the depreciation factor for financial knowledge, δ, but there is little information on the size of this parameter. One study reported that undergraduates' economic knowledge depreciated 4-10 percent annually (Kipps and Kohen 1984). Wage and labor supply information have also been used to measure human capital depreciation; for instance, Heckman (1976) estimated annual depreciation rates of 3-7 percent. We use a value of 6 percent in our baseline calibration and study how results vary from that baseline in the robustness analysis. We could permit the depreciation rate to rise with age to reflect the possibility of cognitive decline. Indeed, this would be consistent with the hump-shaped life cycle pattern of financial savvy reported in Agarwal, Driscoll, Gabaix, and Laibson (2009). Even with a fixed depreciation rate of financial knowledge, however, we show below that we can still produce a hump-shaped financial knowledge profile over the life-cycle.

We also allow for mortality risk differences across education groups, estimated using Gompertz hazard regressions in HRS data for people age 50+, allowing for proportional education effects.[25] We assume the same proportionality by education prior to age 50, but we use age/mortality profiles taken from population life tables.

Upon finding optimal consumption, financial knowledge investment, and technology participation at each point in the state-space and at each age, we then use our decision rules to simulate 5,000 individuals moving through their life cycles. We draw income, out-of-pocket medical expenditure, and rate of return shocks, and we then simulate the life cycle paths of all consumers. These consumers are given the initial conditions for education, earnings, and assets derived from the PSID for individuals age 25-30. We initialize financial knowledge at the lowest level, which is zero. This makes clear how endogenous accumulation of financial knowledge affects wealth outcomes, and it abstracts from differences in initial conditions. A list of the baseline parameters and their values appears in the appendix (and detailed simulation results for all scenarios and calibrations discussed below appear in an online appendix).

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[25]Life expectancy as of age 25 is five years higher for the college-educated compared to high school dropouts. Regression results are available upon request.

6 Results

Our discussion of the simulation results focuses on outcomes around the time of retirement, since this is when heterogeneity in net wealth is most evident. Table 2 compares our simulation results by education group with PSID data, where key outcomes include median wealth, average income, wealth-to-income, the fraction poor (defined as having wealth less than twice income). It also reports the fraction participating in and the share of wealth in the sophisticated technology. The ratio of median wealth-to-income (average lifetime income for each group) is 2.98 for dropouts and 7.3 for the college-educated. In other words, our model generates a strongly positive relationship between accumulated wealth (normalized by income) and income. Moreover, our simulated wealth patterns closely replicate those in the PSID. Specifically, at age 65, the simulated median high school dropout (<HS) has accumulated half as much wealth as high school (HS) graduates ($94,746 versus $177,391), while the College+ group held 3.66 times as much retirement wealth ($346,805) compared to high school dropouts. Using the same average simulated lifetime income measures in the PSID, we obtain a close-to-perfect match between the simulations and the data in terms of normalized wealth (wealth-to-income ratios). Our model generates a College+/<HS median wealth inequality ratio of 2.45, whereas in the PSID it is 2.41. We also simulate the percentage of individuals who are poor in old age, proxied by the fraction of consumers reaching retirement age with assets below twice their incomes. As indicated in Table 2, almost 40 percent of high school droputs are poor according to this metric, versus 17 percent for the College+ group. These results compare closely to the PSID age-65 fraction poor by education, at 35 percent and 16 percent, respectively.

[Table 2 here]

Table 2 further indicates the share of consumers who will invest in the sophisticated technology over their lifetimes, a ratio that also varies by education group. Among the dropouts, 45 percent do so, versus 61 percent of the high school graduates and 78 percent of the College+ group. The PSID pattern is remarkably close to these outcomes, at 28, 63,

and 75 percent, respectively. Generating large differences in participation across education groups is notoriously difficult in models that assume all households can obtain the same returns, if they participate. As we show later, it is the endogenous mechanism of financial knowledge which allows us to generate large differences in participation. Although the model predicts the dispersion in participation rates well, it captures the share of wealth invested in the technology less well. This is because the model predicts that those who use sophisticated saving technologies at retirement will have invested nearly all their assets in it, whereas in the data, the share is closer to 60 percent. Nevertheless, the simulations are consistent with the data in that there is little heterogeneity across education groups in terms of the share invested in the technology. As we show in an extension below, an alternative formulation for preferences allows us to better match the share invested in the technology (Epstein and Zin 1989).

We have also computed the fraction of simulated consumers with low financial knowledge (Low FK) at the time of retirement. Given the production function for knowledge, a threshold of 25 units implies that such households could expect an annual excess return of only one percentage point or less. In our model, such a low level of financial knowledge turns out to be optimal for many, given the constraints and shocks that individuals face. These "optimally ignorant" individuals include 54 percent of the high school dropouts, 37 percent of the high school graduates, and 21 percent of College+. Since financial knowledge strongly influences participation in the sophisticated technology, it is perhaps not surprising that almost all of those with a financial knowledge level of over 25 do use the technology. In this way, financial knowledge can be seen as a type of entry cost, allowing users to deploy the technology effectively. This heterogeneous entry cost is endogenous, and it varies by education group since incentives to invest in financial knowledge also differ.

Figure 4 depicts simulated life cycle paths of average financial knowledge by education groups, all of which prove to be hump-shaped. During the accumulation phase, better-educated consumers invest more in financial knowledge because they have more to gain from higher returns that help them smooth lifetime marginal utilities. At some point, the

opportunity cost of investing becomes too large in terms of foregone consumption and depreciation, and the marginal benefit decreases due to the shorter horizon over which individuals can enjoy the returns from the investments. For these reasons, financial knowledge peaks around age 65 and declines thereafter. This pattern differs from what was found by Agarwal, Driscoll, Gabaix, and Laibson (2009) who report that the age at which people make the best financial decisions is around age 53. One explanation for this difference is that we do not allow for the rate of depreciation to depend on age, for example due to declining cognitive ability.

The present value of average expenditures on financial knowledge over the life-cycle, using a 4 percent discount rate (which is also the preference discount rate), is $9,275 for high school dropouts, $14,442 for high school graduates, and $19,608 for the College+ (in $2004). Undiscounted, these figures are $25,882, $38,397, and $51,954 for the same three groups. Decreasing the depreciation rate of knowledge decreases expenditures on financial knowledge: for instance, with a 3 percent depreciation rate (versus the baseline 6 percent), the present value of expenditures on knowledge drops for each group ($7,758, $11,460, and $14,142, respectively). However with a 9 percent depreciation rate, the present value of expenditures on knowledge is mostly higher than in the baseline ($9,250, $15,031, and $22,458).

[Figure 4 here]

6.1 Quantitative Importance of Endogenous Financial Knowledge

6.1.1 Decomposition Analysis

Our model embodies several differences across education groups that generate differential wealth accumulation patterns. First, the consumption floor acts as a tax on saving for those most likely to experience a substantial negative income shock, since subsistence benefits are means-tested (HSZ, 1995). Second, differences in replacement rates, demographics, and mortality patterns can create differential incentives to save. Finally, there is the mechanism we propose: financial knowledge, which creates a positive relationship between normalized

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wealth and income. To clarify the relative contribution of each mechanism in the life cycle model, we next describe the decomposition exercise depicted in Figure 5.

[Figure 5 here]

To this end, we recall that the ratio of median wealth-to-income for the College+ group to that of high school dropouts was 2.45 at retirement in our baseline (and 2.41 in the PSID data). As a contrast, we next eliminate the possibility of accumulating knowledge along with all differences across education groups other than income while working, as well as medical expenditure differences. For this alternative, we fix all constraints to those of high school graduates and eliminate the consumption floor. The top bar of Figure 5 provides the result: it shows that in a setup with only income and medical expenditure uncertainty, the wealth-to-income ratio of the College+ group is close to that of dropouts, the ratio of the two being 0.87. In other words, confirming what we noted at the outset, the basic life cycle model without extensions predicts that all groups accumulate wealth in roughly the same proportion to income.

Next we reintroduce the consumption floor, which reduces precautionary savings of high school dropouts by more than that of the College+. As illustrated, this does raise the wealth-to-income ratio for the College+ versus high school dropouts, but the impact is small, boosting it to only 0.976 (the second bar in Figure 5). In other words, in our model the consumption floor plays a relatively inconsequential role in generating wealth inequality. This finding differs from HSZ (1995) because our precautionary saving motive is much smaller than theirs due to lower risk aversion.

Re-introducing differences in old-age income replacement rates is important since the college-educated have much lower replacement rates under the Social Security system than do high school dropouts. Moreover, this change alters both wealth accumulation and life-time income patterns; the net effect, of course, depends on the substitutability of retirement wealth and private wealth. The third bar in Figure 5 represents this simulation, which raises inequality by 30 percent (from 0.976 to 1.289). Introducing differences in demographics (the

4th bar down) contributes a smaller increase in the ratio, of 0.13. What this means is that differences in replacement rates in our model are more influential than differences in household composition. Accounting for mortality differences (the 5th bar) again increases the ratio, now to 1.815. This is because college-educated households must finance consumption over a longer horizon, while high school dropouts face a shorter horizon. This is the amount of inequality generated using a life cycle model that lacks endogenous financial knowledge.

The last bar in Figure 5 shows what happens when we reintroduce the possibility of investing in financial knowledge, in addition to the other factors mentioned above. The impact of allowing consumers to access the sophisticated technology and earn higher expected returns is striking. Now the wealth-to-income ratio across education groups rises from 1.815 to 2.45. Thus, of all the explanations examined here for heterogeneity in wealth outcomes, financial knowledge accounts for just over 40 percent of the cross-group wealth inequality [0.406=0.64/(2.45-0.877)].

We have also investigated how far one gets with only initial differences in financial knowledge. To do so, we took differences in financial knowledge at age 25 as reported in the NFCS and shut down endogenous accumulation of financial knowledge over the life-cycle (in this scenario, individuals cannot invest in financial knowledge so the initial differences remain constant over the life cycle). Results show that this adds very little to wealth inequality, suggesting that endogenous financial knowledge is essential if we are to generate the large difference in wealth accumulation we see empirically.[26]

To more fully illustrate the impact of having access to the higher returns as a result of investing in financial knowledge, an additional simple counterfactual exercise is useful. For each education group, we first compute average simulated consumption, investment, and medical expenditures by age. Then we evaluate the average return factor for each education group by age using its accumulated financial knowledge, and we compare this to the average wealth path that would be generated if all groups could only earn the average return earned by high school dropouts. This exercise reveals that wealth would have been 41 percent lower

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[26]Detailed results of this simulation are reported in the online appendix.

for the College+ at the time of retirement had they experienced the returns paid to high school dropouts; for high school graduates, the decline would have been just over 30 percent compared to the paths using their actual average rates of return. Since rates of return differ by roughly 1 percent between education groups, these differences compounded over many years produce substantial differences in wealth holdings. Moreover, our model generates this wealth inequality endogenously building on differences in marginal utilities of consumption over the life cycle.

6.1.2 Alternative Preference Specifications

In this section we offer additional observations regarding the preference structure used in our model. First, we have thus far assumed constant relative risk aversion. By contrast, allowing risk aversion to decrease with wealth could help explain why those with college education invest more in the technology, and decreasing relative risk aversion (DRRA) could then generate additional wealth inequality. Empirical evidence on the relationship between risk aversion and wealth is, however, mixed. On the one hand, Brunnermeier and Nagel (2008) and Chiappori and Paiella (2011) could not reject the assumption of constant relative risk aversion. On the other hand, Calvet and Sodini (2014) found rather compelling evidence of decreasing relative risk aversion using administrative data on twins from Sweden. A common way of modeling DRRA is to assume a subsistence level of consumption, ϕ, such that utility is given by $u(c) = \frac{(c-\phi)^{1-\sigma}}{1-\sigma}$. This subsistence level must be feasible at the lower bound of cash-on-hand. But in a model with uncertain income and medical expenditures, the lower bound for cash-on-hand can be close to zero or negative. This is one reason for having a consumption floor close to that offered by existing transfer programs. Accordingly, to allow for DRRA, we must set the subsistence level below the consumption floor. This lowers the potential for decreasing relative risk aversion to generate high wealth inequality. In addition, because σ governs both risk aversion and the intertemporal elasticity of substitution, a decreasing risk aversion implies increasing intertemporal elasticity of substitution. Hence, the effect of this preference specification on wealth accumulation and wealth inequality is

ambiguous.

Moreover, in a world with CRRA preferences, risk aversion has two effects on people's willingness to invest in the technology. On the one hand, agents are less willing to invest in the technology as risk aversion increases. On the other, they may want to accumulate more wealth for precautionary reasons which could increase their demand for the technology (Haliassos and Michaelides 2003). We conducted two simulations fixing the subsistence level of consumption at 75 percent and 90 percent of the consumption floor set by transfer programs.[27] Compared to the baseline scenario where the observed ratio of normalized wealth between College+ and high school dropouts was 2.45, DRRA lowers wealth inequality to 1.81 and 1.6 respectively. Nevertheless, financial knowledge increased wealth inequality by roughly the same magnitudes in these scenarios, compared to scenarios without financial knowledge but with DRRA.

A second point has to do with how we model preference heterogeneity. Our baseline scenario assumed that preferences are the same across education groups (see the first panel of Figure 6). While preference heterogeneity may offer an alternative explanation for observed behavior, two facts suggest that building in heterogeneity consistent with experimental evidence would reinforce results from our baseline scenario. First, there is mounting evidence that financial knowledge itself is related to returns, risk diversification, and consumption growth (Jappelli and Padula 2013; Calvet, Campbell, and Sodini 2007; Clark, Lusardi, and Mitchell 2014). Hence wealth inequality cannot simply be an expression of preference heterogeneity, though preference heterogeneity could explain wealth inequality through our proposed mechanism of endogenous financial knowledge. Second, the evidence on preference heterogeneity suggests that better-educated households are both more patient (Lawrance 1991; Harrison, Lau, and Williams 2002) and less risk averse (Barsky, Juster, Kimball, and Shapiro 1997; and Kapteyn and Teppa 2011). Accordingly, they would be more inclined to invest in knowledge, which tends to increase rather than decrease the role played by financial knowledge. The second panel of Figure 6 shows what happens in our model when high

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[27]Detailed results are reported in the online appendix.

school graduates have the same preferences as in the baseline ($\beta = 0, 96, \sigma = 1.6$), but the College+ group is allowed to be more patient and less risk averse ($\beta = 0.97, \sigma = 1.4$), and dropouts are more impatient and more risk averse ($\beta = 0.95, \sigma = 1.8$). Compared to the baseline scenario (top panel), wealth inequality is amplified.

[Figure 6 here]

Another interesting feature of our model is the hump-shaped profile of participation in the sophisticated technology. In our model, the young have low wealth and financial knowledge which translates into low participation in the sophisticated technology. This type of behavior is hard to capture in traditional models (for a review, see Guiso and Sodini 2013).

A final observation regards the intertemporal separability of the utility function, and the link between the intertemporal elasticity of substitution and risk aversion. The power utility formulation generates a predicted share of wealth invested in the technology of close to one, which may exaggerate the importance of financial knowledge in explaining wealth inequality. For this reason, we have also considered Epstein-Zin preferences in an alternative scenario by specifying the value function as follows:

$$V_t = \max_{c_t} \left(n_{e,t}(c_t/n_{e,t})^{1-\frac{1}{\psi}} + \beta p_{e,t} E_t[V_{t+1}^{1-\sigma}]^{\frac{1-\frac{1}{\psi}}{1-\sigma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

where ψ is the intertemporal elasticity of substitution and σ is risk aversion. This function collapses to the CRRA function if $1/\psi = \sigma$. We use $\frac{1}{\psi} = 1.6$ and $\sigma = 4$. The choice of ψ is guided by a meta-analysis by Havranek, Horvath, Irsova, and Rusnak (2013) reporting a mean value of 0.6 across 1,429 U.S. studies. The choice of σ is guided by evidence in Barksy, Juster, Kimball, and Shapiro (1997) who found values for $\frac{1}{\sigma}$ close to 0.25. The third panel of Figure 6 displays our results. Not surprisingly, now we find lower shares invested in the technology, driven by a decrease in the conditional shares (share of wealth invested in the technology for those who invest in the technology). As of retirement age, the conditional share is close to 0.6, which matches with what we observe in the PSID (Table 1). Yet the

conditional shares do not vary considerably across education groups, and as a result, wealth inequality remains high. We re-computed results depicted in Figure 5 and conclude that with Epstein-Zin preferences, the share of wealth inequality explained by financial knowledge is still large, namely 33 percent. Hence, although this preference formulation does allow us to better match the share of wealth invested in the technology, it does not affect our key conclusion about the role of financial knowledge in generating an important share of wealth inequality.

In sum, our baseline model using homogeneous CRRA preferences does a relatively good job of accounting for wealth inequality, without having to impose additional preference heterogeneity or more complex preferences. Since these extensions provide rather similar results, we retain our baseline formulation for welfare and policy analysis in the remainder of the paper.

6.2 Optimality

To illustrate the impact of having access to financial knowledge on wealth distributions, we compare what optimal wealth would be at retirement (age 65) in our baseline world where people *can invest* in financial knowledge, with an otherwise identical world where such investment *is infeasible.* To this end, Figure 7 illustrates a scatter plot of simulated wealth targets given the two different environments, for individuals who face the same income shocks and initial conditions. If household wealth holdings were equivalent in the two scenarios, they would lie along the 45-degree line. Conversely, those appearing above the 45-degree line accumulate less wealth when they can invest in knowledge versus not, whereas those below the line do better given access to financial knowledge. The curved line gives a nonparametric plot of the median values of optimal wealth in the two scenarios. The distribution of the cloud of points indicates that having access to financial knowledge generates far more wealth heterogeneity than an economy without such a possibility.

[Figure 7 here]

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The figure provides an interesting perspective for assessing whether households are optimally prepared for retirement. For example, using a model similar to ours but without financial knowledge, SSK (2006) derived optimal wealth targets at the time of retirement. They then examined HRS data and concluded that close to 80 percent of American households accumulated sufficient wealth for retirement, while some 20 percent were under-prepared. By contrast, the wealth distribution in our baseline model closely resembles that observed in the data, suggesting that some proportion of households does not achieve wealth targets as a result of imperfect knowledge. Nevertheless, this is optimal given the constraints they face. Moreover, those who might appear to be "ill-prepared" in our simulations are the less-educated and least financially knowledgeable, usually as a result of large income shocks experienced early in the life cycle. Again, this pattern conforms to our model's theoretical predictions.

Table 3 offers a different perspective, namely a comparison of our baseline scenario key outcomes with those from a counterfactual where all consumers are endowed with perfect financial knowledge at the outset. In particular, we again simulate outcomes for individuals who face the same income shocks and initial conditions; the top panel replicates our baseline, while the lower panel endows everyone with complete financial knowledge when they enter the labor market. We compare these two outcomes using a welfare measure that evaluates the percentage change in permanent consumption that a consumer in our baseline would need in order to make him as well off as in the world with perfect knowledge. For high-school dropouts, the change in lifetime permanent consumption is around 1.9 percent, whereas for the two more educated groups, lifetime consumption would be 2.7 percent higher.

[Table 3 here]

6.3 Varying Parameters for Preferences, Knowledge Costs, Depreciation, and Financial Knowledge Production

We next provide a rich set of sensitivity analyses to help assess how results might change when important parameters are varied. Table 4 provides results for the baseline case (in the

first row), as well as outcomes in which we vary one parameter at a time. In all cases, we report simulated outcomes for the high school dropouts (first column), along with the ratio of simulated outcomes at retirement for the College+ versus the high school dropouts. The second column uses wealth-to-income values at retirement; the third reports the fraction investing in the sophisticated technology at retirement; and the last depicts the fraction with low financial knowledge at retirement. (Additional simulation results for parameters in Table 4 appear in the online appendix.)

[Table 4 here]

6.3.1 Preferences

We first vary the relative risk aversion parameter σ from 1.2 to 2.0 (spanning the baseline value of 1.6). Not surprisingly, this has an important impact on resulting wealth inequality, as well as on the fraction investing in the sophisticated technology and the fraction with low knowledge. Specifically, when risk aversion is low ($\sigma = 1.2$), wealth of the dropouts falls and inequality is amplified relative to the baseline. When the intertemporal elasticity of substitution is large, individuals are more sensitive to rates of return. Better-educated people are willing to invest in financial knowledge and defer consumption. Such complementarity substantially increases the dispersion of wealth across educational groups, such that the degree of inequality increases by about one-third [36%=(3.332/2.45)-1]. In contrast, setting σ to 2 produces the opposite result: the precautionary saving motive is now more important, so all groups accumulate more wealth and therefore have added incentives to invest in financial knowledge and participate in sophisticated products. Accordingly, in a cross-sectional context, our model implies that more risk averse individuals would invest more in financial knowledge and are more likely to use the sophisticated technology. This feature, which is present in a number of prior papers (e.g. Gomes and Michaelides 2005), has to do with the fact that raising the degree of risk aversion typically boosts precautionary wealth and pushes individuals over the participation threshold. This effect dominates the direct effect of risk aversion on risk-taking, which would imply less participation in the so-

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phisticated technology. Overall, higher risk aversion reduces wealth inequality and reduces the correlation between education and financial knowledge.

Considering time preference, we find that wealth inequality is relatively insensitive to changes in the discount factor, β, set initially to a value of 0.96.

6.3.2 Cost of Investing in Knowledge

The next two lines in Table 4 illustrate that results change relatively little when we vary the depreciation rate for financial knowledge, δ, from the baseline level of 0.06 to a low of 0.03 and a high of 0.09 (holding other factors constant). Wealth inequality changes are small, on the order of 2 percent. The higher depreciation rate generates less wealth heterogeneity because the College+ group cuts back on financial knowledge investments, and relatively more in this set fall into the low knowledge category than at baseline.

The next four rows indicate how results depend on the production function for financial knowledge, $\pi(i)$. It will be recalled that the baseline representation of this function $\pi(i_t) = 50i_t^{1.75}$ had two parameters: the multiplicand affects the average cost of acquiring financial knowledge, while the exponent influences the function's convexity. We vary both in Table 4, in turn. Varying the multiplicand from its baseline value of 50 to a low of 25 and a high of 75 has a relatively small impact on the results. By contrast, changing the function's convexity has a non-linear effect. Increasing convexity gives larger incentives to spread investment over the life cycle and to avoid large investments; hence, this should lower differences in financial knowledge. Nevertheless, raising convexity also increases the average cost of reaching a certain level of financial knowledge, which could amplify differences as college-educated households have more resources. The net effect we observe is not monotonic, as inequality first rises (from 1.942 to 2.45) and then falls slightly (from 2.45 to 2.446). The next two rows of Table 4 change the fixed cost of participation in the sophisticated technology, c_d. Varying c_d around the baseline value of $750 has relatively little impact on wealth-to-income ratios, along with the other two outcomes.

6.3.3 Production of Financial Knowledge

To further assess how the financial knowledge production function can shape wealth dispersion, we next examine a different production from that in the baseline setup. Consider the following function:

$$r(f_{t+1}) = \alpha_0 f_{t+1}^{\alpha_1}.$$

Here α_1 is the elasticity of risk-adjusted returns to financial knowledge. We lack evidence on rates of return at the household level, since little is known about this elasticity. Instead, we use a plausible range from the human capital literature (Browning, Hansen, and Heckman 1999), namely 0.5 to 0.9, and we consider three elasticity values: low ($\alpha_1 = 0.5$), medium ($\alpha_1 = 0.75$), and high ($\alpha_1 = 0.9$). The final three rows of Table 4 report the ratio of the College+ to <HS median wealth for each of these scenarios. The results show that when the production function is more concave, less dispersion in income-to-wealth ratios is generated. For example, a high elasticity of 0.9 leaves inequality virtually unchanged (2.43). An elasticity of 0.75, at the mid-range of the human capital literature, yields a ratio of 2.45, while a lower-bound elasticity of 0.5 yields a value of 2.351. It is worth noting that even our lowest value of 2.351 represents considerably more wealth inequality than results from a model without financial knowledge (1.815). Accordingly, more concavity in the knowledge production function attenuates the wealth inequality created by financial knowledge. Allowing for a concave production function with plausible elasticities instead of a linear function still implies a substantial role for financial knowledge.

An alternative to our approach to investment in financial knowledge is a "learning-by-doing" formulation, where individuals can only invest in knowledge according to the following process: $f_{t+1} = (1 - \delta)f_t + \phi I(\kappa_t > 0)$. Figure 8 plots profiles for two values of ϕ : (5,10). We cannot distinguish learning-by-doing from direct investment in financial knowledge empirically, as that would require nonexistent panel data on financial knowledge and portfolio choices. Nevertheless, it is clear that once again, wealth inequality is strongly influenced by financial knowledge, and the resulting patterns of participation in the sophisticated tech-

nology and median wealth patterns are again consistent with empirical patterns depicted above. Additionally, this formulation does not generate as much inequality as in our baseline formulation.

[Figure 8 here]

In a final sensitivity analysis regarding the production function, we briefly explore the impact of allowing financial knowledge to not only raise the expected return on the sophisticated technology, but also to lower the variance of returns. Although more educated individuals may take more risk by investing more in sophisticated products, they may be able to better diversify their portfolios, thus reducing unsystematic risk. The baseline specification captures the first aspect (risk taking increases with education), but it does not capture the second: diversification. To assess the latter's likely importance, we set the standard deviation of returns as $\sigma(f_t) = \sigma_{max} - \theta f_t$ with $\sigma_{max} = 0.25$ with θ fixed such that $\sigma(100) = 0.16$. Hence, an investor without knowledge would do significantly worse than the market, perhaps by picking individual stocks. As he becomes perfectly knowledgeable, his portfolio achieves the same degree of diversification as the market index. Not surprisingly, participation in the technology and wealth is higher for all groups, yet wealth inequality increases only slightly. The ratio of normalized wealth increases slightly to 2.52, versus 2.45 in the baseline scenario (Figure 9).

[Figure 9 here]

In sum, the sensitivity analyses confirm that treating financial knowledge as an investment in human capital generates wealth inequality consistent with the data. The quantitative importance of financial knowledge for wealth inequality does vary somewhat with parameter values, but the amount of variability in inequality explained by endogenously-generated financial knowledge remains high across a wide range of parameter values and processes. Figures 8 and 9 again both illustrate a hump-shaped profile of participation over the life cycle, which is a noteworthy feature of our model and is robust to alternative specifications of the productive function.

6.4 Policy Simulations

In the real world, several institutional factors can help shape the process of financial knowledge accumulation. For instance, Social Security benefits may crowd out household saving and also discourage the accumulation of financial knowledge. Similarly, means-tested benefits protect consumers against bad states of nature and reduce the need to save: having such programs may provide a disincentive to invest in financial knowledge. To explore the relative importance of each, we next undertake two policy simulations and compare results to the baseline findings. In the first case, we examine the impact of a reduction in means-tested benefits by half, which could mean either that generosity is decreased or that eligibility is restricted. In the second case, we reduce expected retirement benefits by 20 percent, reflective of what the Social Security system may be able to pay future retirees unless program revenues are substantially increased (Cogan and Mitchell 2003). Results appear in Table 5, where the top panel reproduces baseline results for ease of comparison.

[Table 5 here]

Our first scenario halves the means-tested consumption floor from $10,000 to $5,000 per year. As is clear from the second panel of Table 5, this boosts incentives to save for precautionary reasons. But because our precautionary saving motive is less important than in other studies, such a policy change does not generate large effects in terms of wealth accumulation or financial knowledge. That is, both wealth and knowledge rise following the benefit reduction, but the impact is relatively similar across education groups. Accordingly, in this model, means-tested benefits do not appear to be an important factor shaping saving and investment in financial knowledge.

An alternative scenario reduces Social Security income benefits by one-fifth, and this does produce a substantial increase in wealth for all educational groups compared to the baseline (see the third panel of Table 5). Median assets rise by one-third (32 percent) for dropouts, and they rise by roughly one-fifth (19 percent) for the College+. Lowering retirement income generosity thus reduces wealth inequality, instead of increasing it. We can also compute

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the change in the present value of retirement income by education group in this scenario. Expressing the change as a fraction of the change in the expected present value of retirement income yields an estimate of the displacement or crowd-out effect of retirement income. A simple life cycle model would predict a complete offset once adjustment is made for the fact that wealth is measured at the time of retirement, so the reduction in lifetime income is only partially offset by that age. By contrast, the unadjusted displacement effects in our simulations range from -0.784 to -1.15.[28] The percentage of dropouts who face shortfalls due to having assets below twice their current income falls by 25 percent, with an even larger change among the best-educated. All groups boost their holdings of the sophisticated technology, and even more interestingly, the fraction of optimally ignorant respondents falls. In other words, since all consumers must now save for retirement, investment in financial knowledge rises across the board. Of course, this comes at a cost: the present value of investment expenditures rises by about $1,000 for dropouts, and $2,000 for high school and the college-educated.[29]

In sum, we have shown that the economic environment affects investment in knowledge, which in turn drives wealth accumulation patterns.[30] Moreover, a learning-by-doing approach leaves our results qualitatively unchanged, though it does not match the data as well as our preferred specification.

7 Discussion and Conclusions

This paper has developed an augmented stochastic life cycle model that endogenizes the decision to acquire financial knowledge, so as to explore the forces that shape financial

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[28]These numbers are computed as the change in wealth at retirement divided by the change in the present value of retirement income when we reduce the generosity of pensions. We use a discount rate of 4 percent for these calculations.

[29]These present values are computed using simulated data on financial knowledge investment over the life cycle, the cost function, and a discount rate of 4 percent. The online appendix presents additional results for Table 5.

[30]In a more realistic context which allowed for heterogeneity in the cost of acquiring knowledge, some individuals might not react to a reduction in retirement benefits by investing in financial knowledge and accumulating more wealth. For this reason, our policy simulations should be interpreted keeping this caveat in mind.

knowledge accumulation over the lifetime and to evaluate how much wealth inequality might be attributable to differences in financial knowledge. Our formulation posits that people can invest in sophisticated financial technology generating higher expected returns, though it is costly to acquire and depreciates with time. Most importantly, we show that allowing for endogenous financial knowledge generates large differences in wealth holdings: specifically, we find that 30-40 percent of U.S. wealth inequality can potentially be attributed to financial knowledge. The profile of optimal financial knowledge proves to be hump-shaped over the life cycle, and it also differs by educational groups because of differences in life cycle income paths. Accordingly, our model can also account for a sizable share of observed differences in wealth across education groups, while other authors have had to rely on heterogeneous preference patterns or means-tested social programs to generate wealth dispersion.

In generating wealth inequality above and beyond what traditional models of saving have delivered, we rationalize some of the large differences in wealth found in much prior empirical work on saving. Our results do not rely on individuals having misperceptions about future returns or other behavioral biases. Instead, our model rests on the important and intuitively sensible fact that individuals do not start their economic lives with full financial knowledge; rather, financial knowledge is acquired endogenously over the life cycle. Moreover, the model does not require differential abilities to acquire financial knowledge or different preferences, so this parsimonious parameterization helps clearly indicate the contribution of endogenous financial knowledge to wealth inequality. We also show that some level of financial ignorance may actually be optimal: inasmuch as it is expensive to acquire financial knowledge and not everyone benefits from greater financial sophistication, some consumers will rationally remain financially ignorant.

Incorporating this quite-realistic mechanism can also yield interesting policy predictions. For instance, nations promising higher levels of old-age benefits would be expected to be those with lower levels of financial knowledge in the population, while growing reliance on individually managed 401(k) accounts should be accompanied by rising financial knowledge. Moreover, the model can help explain low levels of financial knowledge around the world,

and it also rationalizes why some population sub-groups are ill-informed, particularly those anticipating larger old-age social insurance benefits. It also offers insights about the potential effects of adding financial education programs in high school or the workplace, as these can be modeled as lowering the cost of, or increasing the endowment of, financial knowledge (naturally, a full analysis would also require the measurement of program costs). And the model can also inform policymakers regarding the timing of financial education over the life cycle, in that the benefits of a longer horizon must be compared against the costs of acquiring financial knowledge early, when the marginal utility of consumption is higher and households are more likely to face liquidity constraints. Finally, our model helps explain why financial education programs might not produce large behavioral changes, particularly for people who find it suboptimal to invest in financial knowledge. That is, a policy intended to raise financial knowledge early in life might not have measurable long-term effects, if consumers have both optimal financial knowledge and optimal target wealth levels in mind when solving their life cycle problems. In other words, offering financial knowledge can boost saving in the short run, but it might have little enduring impact in terms of boosting lifetime wealth. In sum, incorporating endogenous financial knowledge into a life cycle model has important implications for the economic understanding of how much consumers save and invest over their lifetimes, as well as how they will invest.

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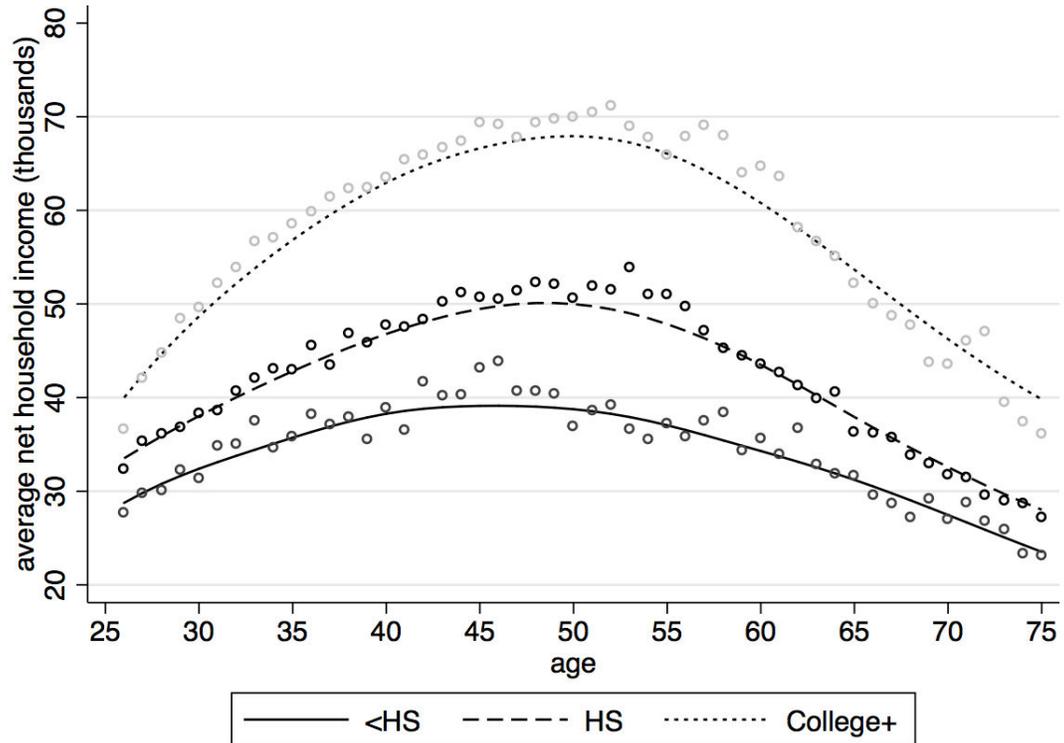
Figure 1: **Life Cycle Net Household Income Profiles by Educational Attainment**.
This figure shows median net household income by education group computed from the
PSID for waves 1984-2005 (in $2004; see text). The term <HS refers to households where
the head has less than a high school diploma, HS indicates the head completed high school,
and College+ means the head had at least some college. The figure adjusts for cohort effects
based on median regressions with age controls; predictions are for those born 1935-1945.
The income measure excludes capital income, and observations with income in excess of
$250,000 are also excluded. Age profiles are smoothed using a lowess filter.
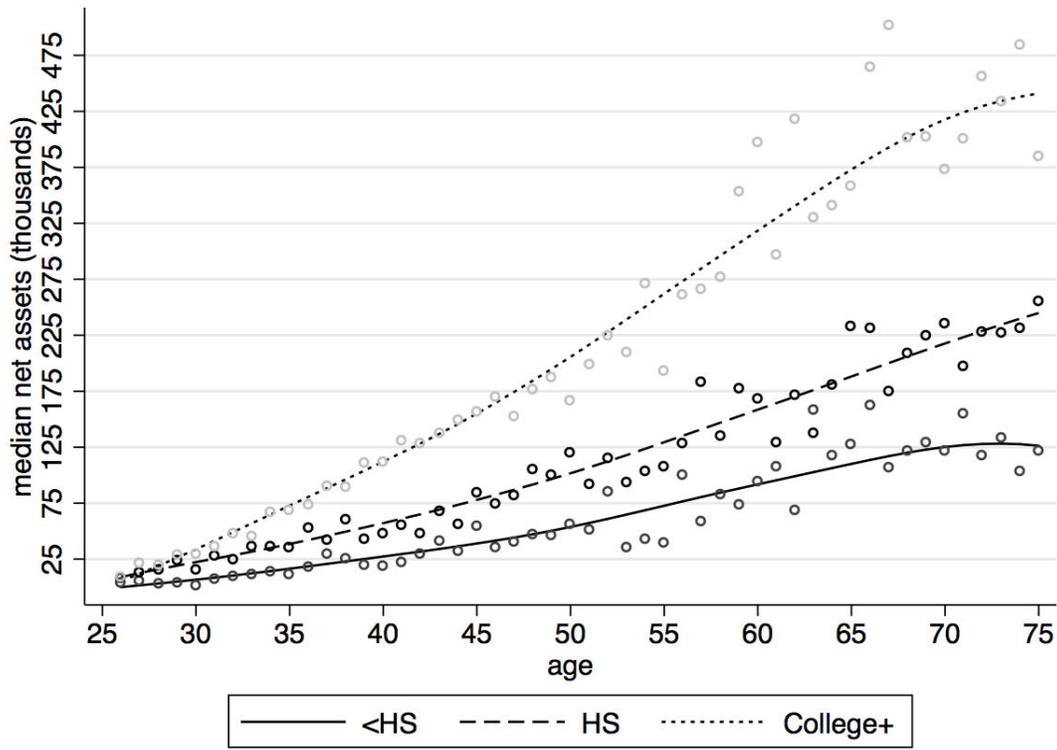
Figure 2: **Life Cycle Wealth by Educational Attainment**. This figure reports median wealth profiles by education group, from the PSID (in $2004; see Figure 1 and text). The profiles are predicted from median regressions with a correction for cohort effects (following French, 2005); wealth refers to the sum of assets minus debt. Wealth is predicted for all persons born 1935-1945; age profiles are smoothed using a lowess filter.
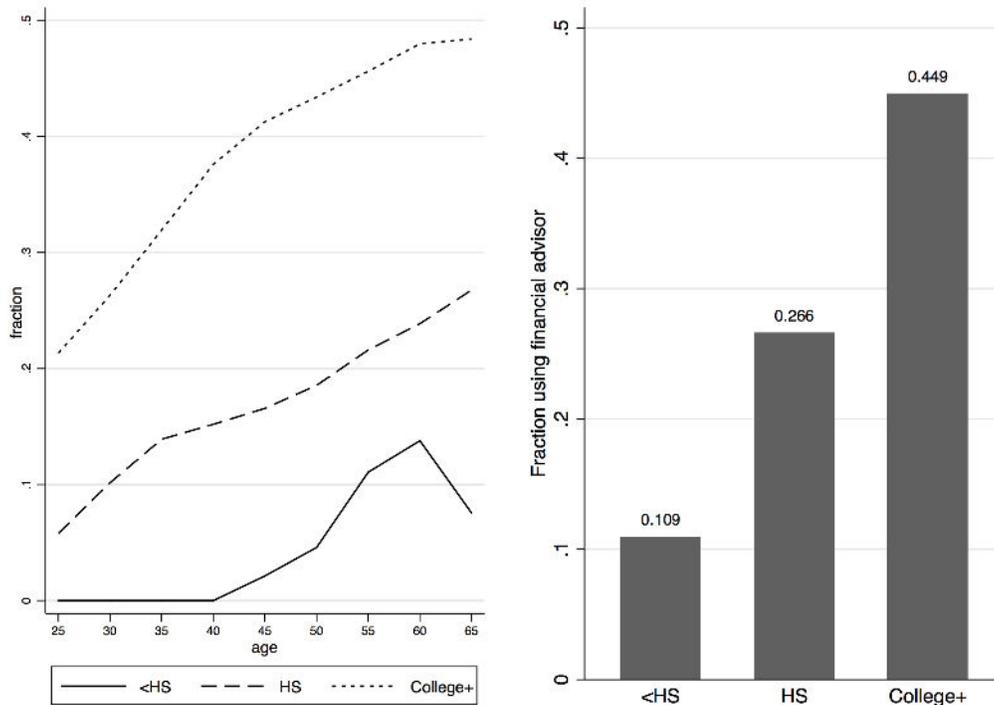
Figure 3: **Observed Financial Knowledge and Use of Financial Advice by Age and Education.** The left panel of this figure shows the fraction of respondents in the 2012 National Financial Capability Study (NFCS) who answered all five financial knowledge questions correctly, by five-year age groups and three education levels. The right panel shows the fraction of respondents in the NFCS, by education level, who reported using a financial adviser. See also Figure 1.
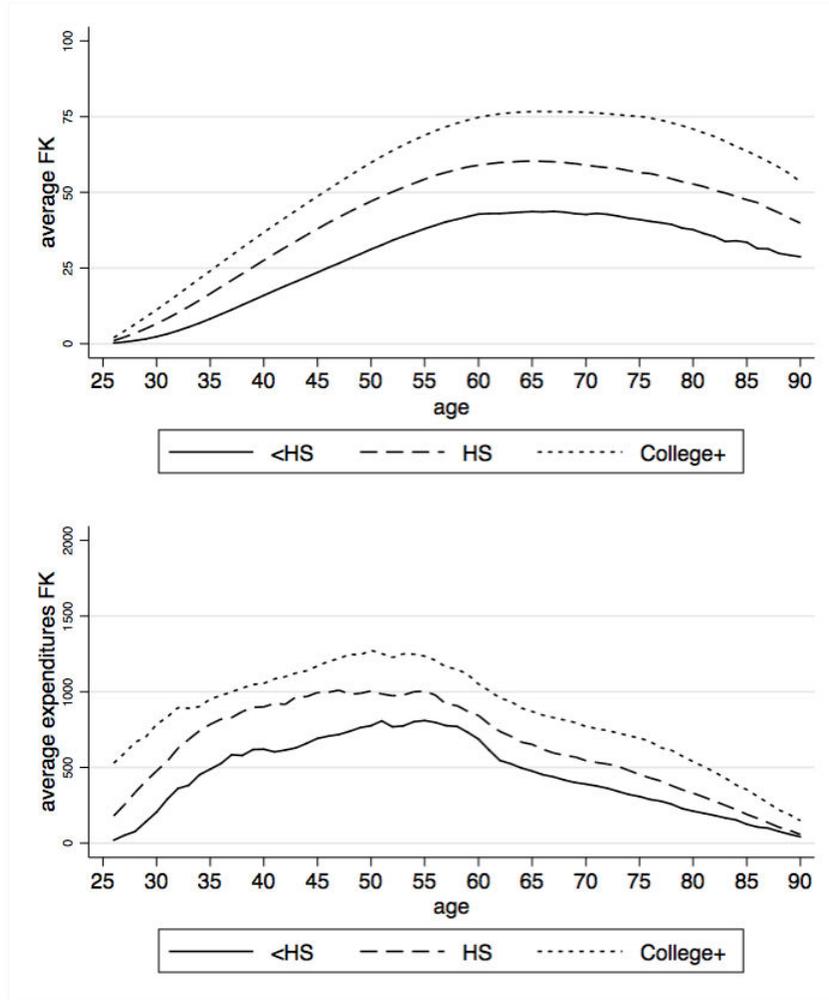
Figure 4: **Simulated Average Levels of and Expenditures on Financial Knowledge**
(FK). These figures report average levels of financial knowledge f_t as well as average annual
expenditures on financial knowledge $\pi(i_t)$, by age and education levels. See also Figure 1.
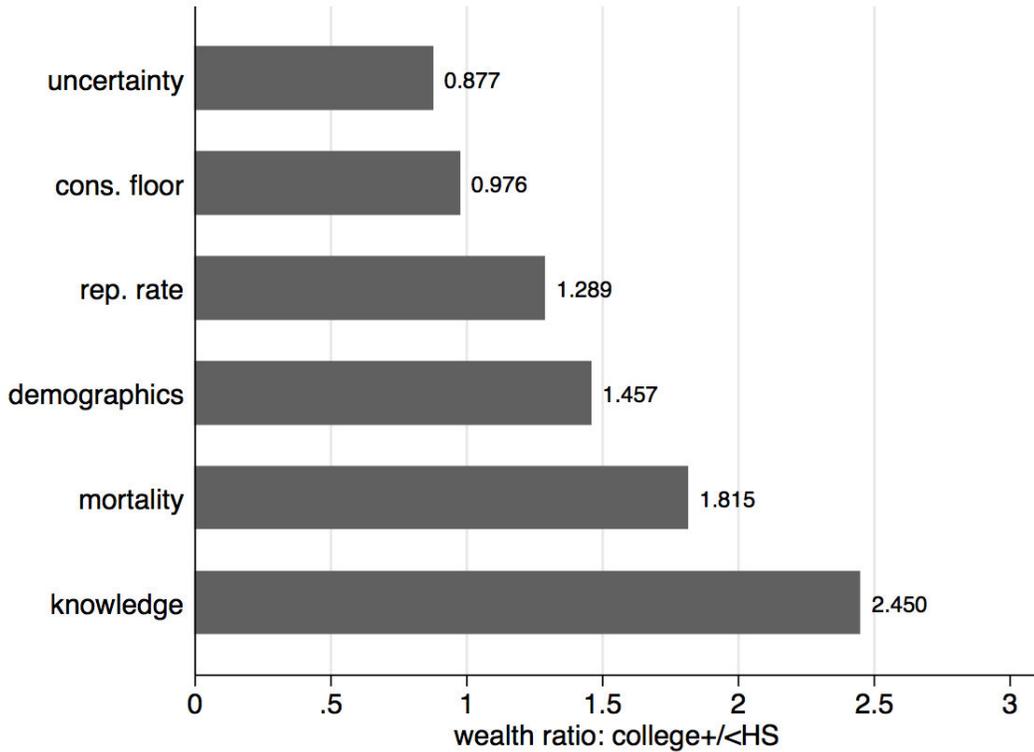
Figure 5: **Decomposition of Wealth Inequality at Retirement**. This figure reports simulated median wealth divided by average lifetime income, expressed as a ratio of the College+ to high school dropouts (<HS). The top bar accounts only for differences in uncertain lifetime income and medical expenditures; all other sources of heterogeneity are set to the parameters of those who completed high school (HS). Subsequent bars then progressively add mechanisms that can generate dispersion in wealth-to-lifetime average income ratios. The second bar adds a consumption floor. For the third bar, we add differences in replacement rates by education group. The fourth bar includes differences in household size over the life cycle. The fifth bar incorporates mortality differences by education. The final bar adds the impact of financial knowledge accumulation. For comparison, the PSID wealth-to-income ratio (Table 2) is 2.45. See Figure 1 and text for definitions. (Additional detail is provided in the online appendix.)
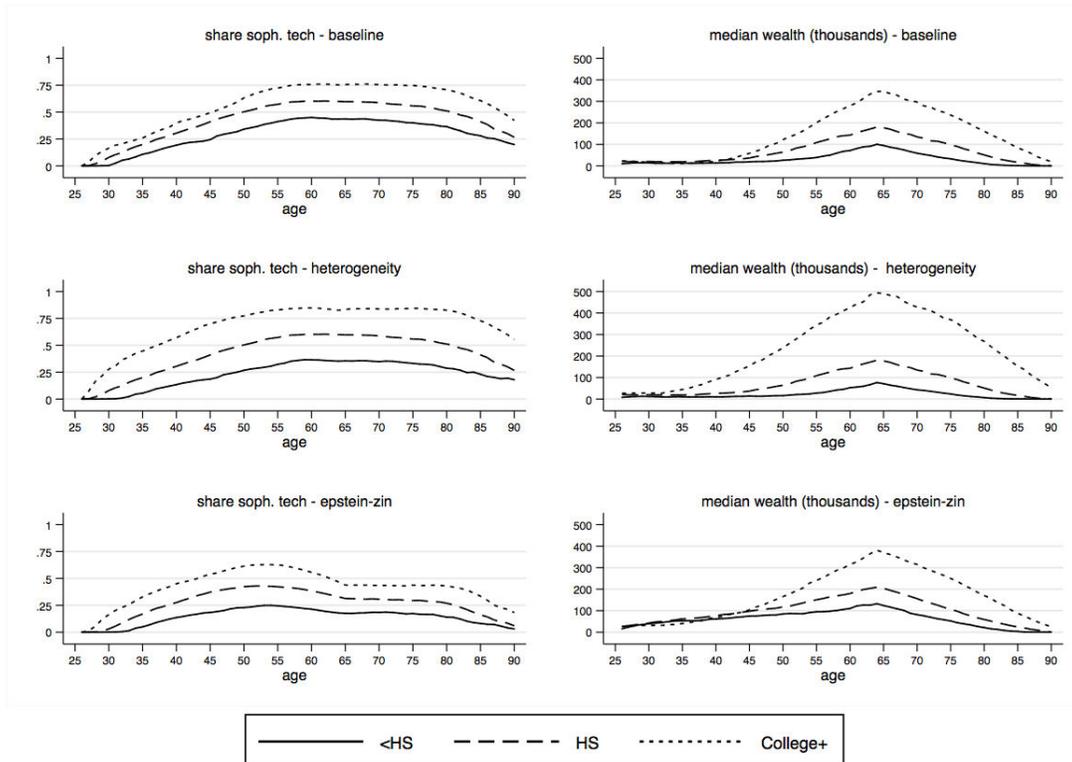
Figure 6: **Simulated Life-Cycle Wealth and Fraction of Wealth invested in the Sophisticated Technology with Alternative Preference Specifications**. These figures trace the share of wealth invested in the technology and median wealth by age and education under three scenarios. The top panel refers to our baseline scenario; the middle panel allows for heterogeneity in preferences; and the bottom panel assumes Epstein-Zin preferences. See also text and Figure 1 for definitions. (Additional detail is provided in the online appendix.)
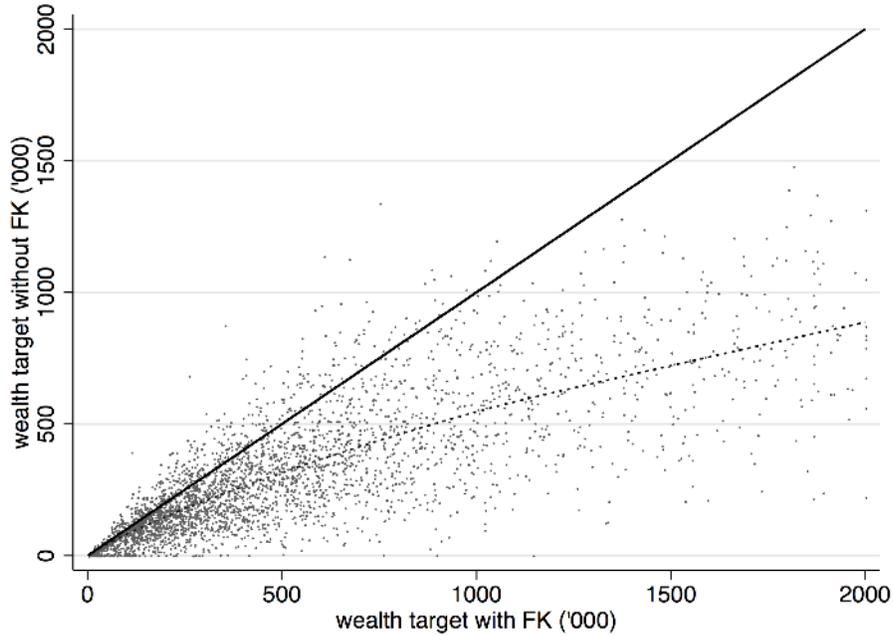
Figure 7: **Comparison of Simulated Predicted Wealth at Retirement: Base Case and Scenario Without Financial Knowledge (FK).** This figure reports simulated wealth targets at retirement (age 65, in $000) under two scenarios for the same individuals (same income shocks and initial conditions). The first scenario (wealth target with FK) is generated using the baseline where individuals can invest in financial knowledge if it is optimal to do so. The second scenario (wealth target without FK) assumes individuals cannot invest in financial knowledge. Each dot represents a pair of simulated wealth targets. The 45-degree line is also plotted as well. Individuals above the 45-degree line have accumulated less wealth under the FK scenario than under the scenario without FK, and vice versa. We also plot a non-parametric estimate of the relationship between the two targets (dotted line).

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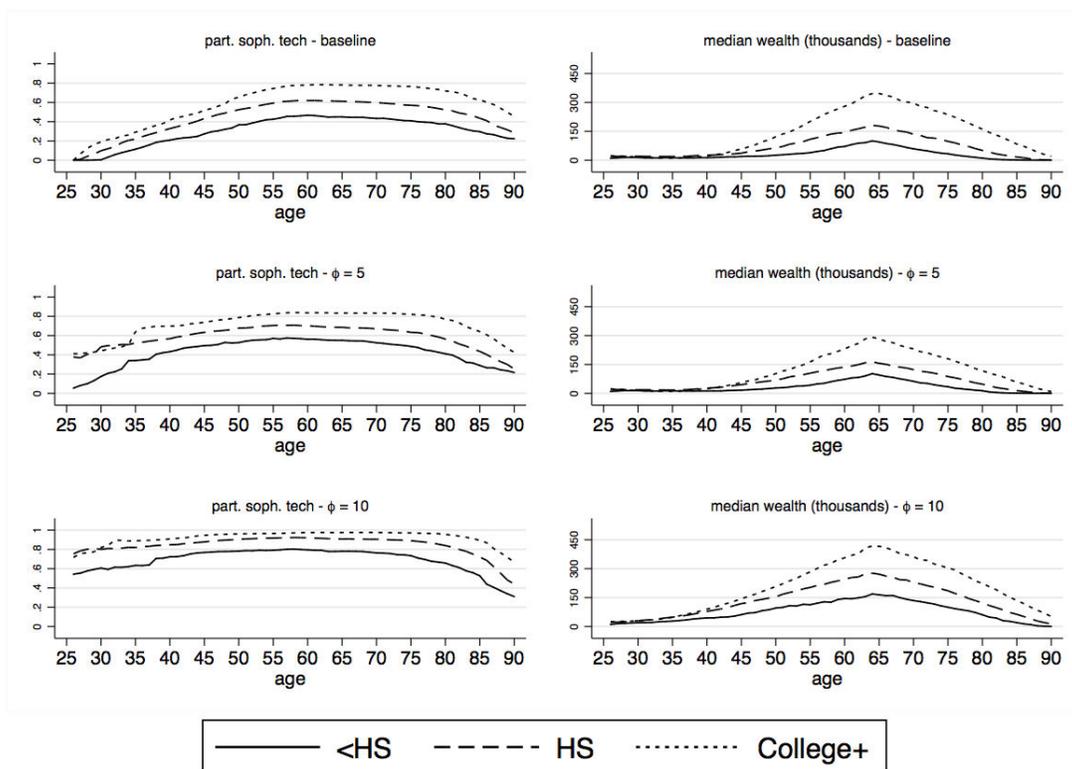
Figure 8: **Simulated Life-Cycle Wealth and Fraction Investing in the Technology with Learning-by-Doing**. These figures report the simulated life-cycle profiles of median wealth and the fraction investing in the technology by level of education, for individuals who can invest only in their financial knowledge using a learning-by-doing technology. The accumulation of knowledge follows from $f_{t+1} = \delta f_t + \phi I(\kappa_t > 0)$. Profiles are plotted for two values of ϕ : (5,10). See also text and Figure 1 for definitions.
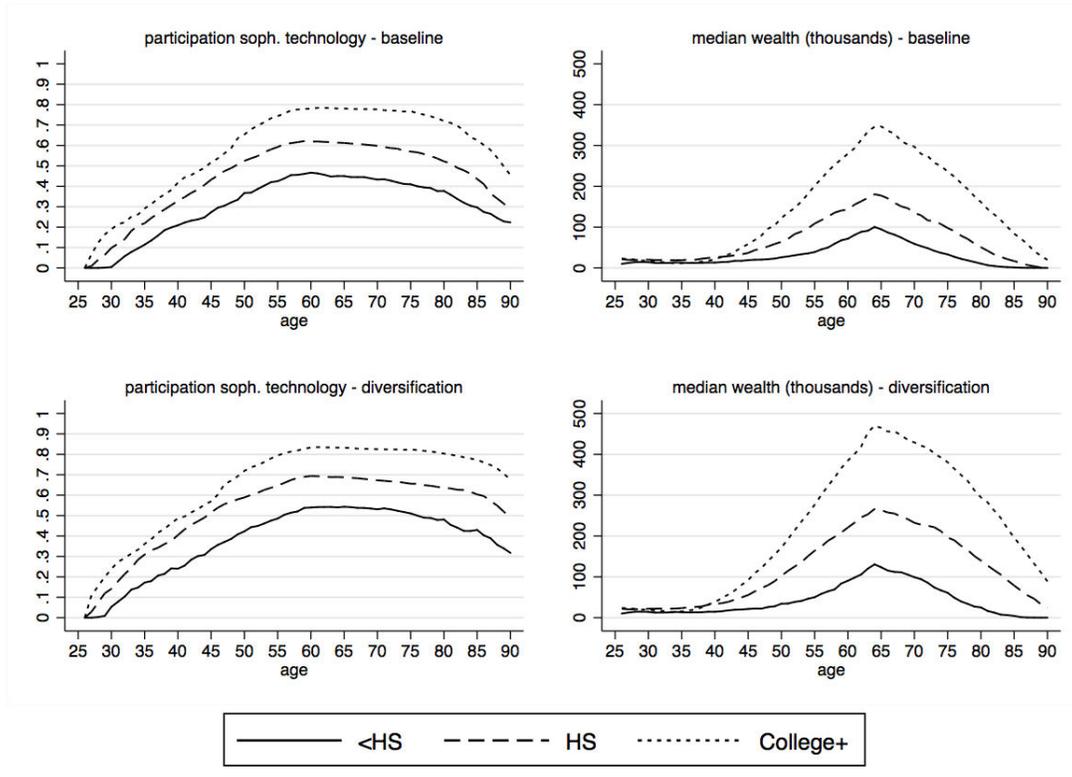
Figure 9: **Life Cycle Simulated Path with Diversification Benefits to Investing in Financial Knowledge**. These figures report the simulated life-cycle profiles of participation in the technology and median wealth, in a scenario where financial knowledge not only raises the expected return on the technology but also lowers the variance of returns. The standard deviation of returns is given by $\sigma(f_t) = \sigma_{max} - \theta f_t$ with $\sigma_{max} = 0.25$ and θ is fixed such that $\sigma(100) = 0.15$. See also text and Figure 1 for definitions.

	<HS	HS	College +	Total
By Age		*Participation Rate*		
25-35	.158	.177	.541	.369
35-45	.173	.331	.623	.477
45-55	.193	.415	.651	.54
55-65	.28	.522	.752	.635
65-75	.421	.586	.732	.622
Total	.243	.39	.652	.52
By Age		*Share Financial Wealth*		
25-35	.391	.318	.442	.416
35-45	.393	.435	.516	.493
45-55	.478	.486	.559	.539
55-65	.5	.508	.564	.547
65-75	.584	.484	.611	.561
Total	.502	.468	.543	.522

Table 1: **Life Cycle Participation and Share of Portfolio in Sophisticated Financial Products (Stocks and IRAs) by Educational Attainment in the PSID** (2001-2005). This table reports participation percentages and the share of financial wealth invested in sophisticated financial products from PSID data (see text). The term <HS refers to households where the head has less than a high school diploma, HS indicates the head completed high school, and College+ means the head had at least some college.

	<HS	HS	College +	Ratio (College+/<HS)
Simulation				
Median wealth	94746	177391	346805	3.66
Average Income	31780	38703	47485	1.494
Wealth-to-Income Ratio	2.981	4.583	7.303	2.45
Fraction Poor ($a_t < 2y_t$)	.387	.2903	.1742	.4501
Participation ($\kappa_t > 0$)	.4502	.6123	.7811	1.735
Share $E[\kappa_t|\kappa_t > 0]$	.9712	.9748	.9687	.9975
Low FK ($f_t < 25$)	.5382	.3665	.2092	.3888
Data (PSID)				
Median wealth	101872	193392	365392	3.587
Wealth-to-Income Ratio	3.21	4.99	7.695	2.40
Fraction Poor ($a_t < 2y_t$)	.3478	.3039	.1588	.4565
Participation ($\kappa_t > 0$)	.2817	.6289	.7538	2.676
Share $E[\kappa_t|\kappa_t > 0]$	.5834	.5231	.5787	.9919

Table 2: **Simulated and Observed Outcomes at Retirement (age 65).** This table summarizes outcomes from baseline simulations at age 65 compared to actual observed outcomes in the PSID. The last column shows the ratio of College+ to high school dropout values for each row ($2004). Average income is average simulated lifetime income. A poor household is defined as a household which has less than twice its income in accumulated wealth. Participation denotes the fraction who invest in the technology and the share denotes the average share of wealth invested in the technology for those who invest in the technology. Finally, those with low financial knowledge are those with less than 25 units of financial knowledge. (Additional detail is provided in the online appendix.)

	<HS	HS	College +	Ratio (College+/<HS)
Baseline				
Median wealth	94746	177391	346805	3.66
Wealth-to-Income Ratio	2.981	4.583	7.303	2.45
Fraction Poor ($w_t < 2y_t$)	.387	.2903	.1742	.4501
Participation ($\kappa_t > 0$)	.4502	.6123	.7811	1.735
Perfect FK				
Median wealth	173150	282963	436800	2.523
Wealth-to-Income Ratio	5.448	7.311	9.199	1.688
Fraction Poor ($w_t < 2y_t$)	.2525	.1429	.07364	.2917
Participation ($\kappa_t > 0$)	.7919	.8865	.9282	1.172
Welfare (% baseline $\bar{c}$)	0.019	0.027	0.027	

Table 3: **A Comparison of Baseline Results versus Those From a Model with Perfect Financial Knowledge (FK).** This table compares simulated outcomes from the baseline scenario, compared to a model in which households are endowed with complete financial knowledge at the point of entering the labor market. Wealth-to-Income ratio denotes the ratio of median wealth to average lifetime income. A household is defined as poor if it has accumulated less than twice its income at the time of retirement. The welfare measure evaluates the percentage change in permanent consumption that would be equivalent to the change in expected utility at age 25 under the perfect knowledge scenario. (Additional detail is provided in the online appendix.)

	Level (<HS)	Ratio (College+/ <HS)		
	Wealth-to-Income	Wealth-to-Income	Participation	Low FK
Baseline	2.98	2.45	1.735	0.388
$\sigma = 1.2$	2.02	3.332	1.851	.4182
$\sigma = 2.0$	3.75	2.118	1.648	.3633
$\beta = 0.95$	1.99	2.308	1.904	.528
$\beta = 0.97$	4.50	2.249	1.502	.2785
$\delta = 0.03$	3.33	2.285	1.478	.2994
$\delta = 0.09$	2.50	2.509	1.986	.4884
$\pi_0 = 25$	3.65	2.215	1.458	.2777
$\pi_0 = 75$	2.50	2.487	2.05	.4702
$\pi_1 = 1.25$	4.32	1.942	1.427	.2362
$\pi_1 = 2$	2.49	2.446	2.079	.4628
$c_d = 1000$	2.82	2.487	1.811	.4163
$c_d = 500$	3.04	2.456	1.61	.3698
$\alpha_1 = 0.9$	3.01	2.43	1.679	.3824
$\alpha_1 = 0.75$	3.02	2.45	1.547	.3936
$\alpha_1 = 0.5$	3.18	2.351	1.471	.3581

Table 4: **Sensitivity Analysis for Preference and Cost Parameters and Production Function.** Each row reports the ratio of College+ to <HS outcomes as of age 65, for the case when we vary the single parameter indicated versus the baseline case. Baseline values as reported in the text are: relative risk aversion ($\sigma = 1.6$), financial knowledge depreciation rate ($\delta = 0.06$), investment production function ($\pi(i) = 50i^{1.75}$), participation cost ($c_d = 750$), and discount factor ($\beta = 0.96$). In the final three rows we alter the elasticity of returns to financial knowledge (α_1), which is implicitly equal to 1 in the baseline. The first column reports the ratio of normalized median wealth of the College+ group to the median of the <HS group. Both are normalized by average lifetime income. The Participation Ratio denotes the fraction of College+ individuals participating in the technology divided by the same fraction for the <HS group. The final column reports the same ratio for only those individuals with fewer than 25 units of financial knowledge (Low FK). See Table 1 and text for definitions. (Additional detail is provided in the online appendix.)

	<HS	HS	College +	Ratio (College/<HS)
Baseline				
Median wealth	94746	177391	346805	3.66
Wealth-to-Income Ratio	2.981	4.583	7.303	2.45
Fraction Poor ($a_t < 2y_t$)	.387	.2903	.1742	.4501
Participation ($\kappa_t > 0$)	.4502	.6123	.7811	1.735
Low FK ($f_t < 25$)	.5382	.3665	.2092	.3888
Reduced Floor ($c_{\min}$)				
Median wealth	108783	191395	360725	3.316
Wealth-to-Income Ratio	3.423	4.945	7.597	2.219
Fraction Poor ($a_t < 2y_t$)	.3555	.271	.1585	.4458
Participation ($\kappa_t > 0$)	.4655	.6181	.766	1.645
Low FK ($f_t < 25$)	.515	.3492	.192	.3728
Reduced Retirement Income				
Median wealth	125194	235167	412369	3.294
Wealth-to-Income Ratio	4.077	6.302	9.011	2.21
Fraction Poor ($a_t < 2y_t$)	.2907	.1771	.08989	.3092
Participation ($\kappa_t > 0$)	.485	.6613	.798	1.645
Low FK ($f_t < 25$)	.485	.2997	.1549	.3194

Table 5: **Simulation Results of Policy Experiments**. This table summarizes outcomes from simulations at retirement age. The first panel replicates our baseline scenario; the second lowers means-tested benefits from $10,000 to $5,000; and the third lowers retirement income by 20 percent. A poor household is defined as one with less than twice its income in accumulated wealth. Participation denotes the fraction who invest in the sophisticated technology. Those with low financial knowledge are those with less than 25 units of financial knowledge. See text; additional details are provided in the online appendix.

Appendix

Parameter	Definition	Value
σ	relative risk aversion	1.6
β	discount factor	0.96
$\overline{r}$	safe return	0.02
σ_ϵ	standard deviation returns	0.16
$r(f_{max})$	maximum excess return	0.04
π_0	productivity of investment function $(\pi(i_t) = \pi_0 i_t^{\pi_1})$	50
π_1	concavity of investment function	1.75
c_d	participation cost	750
δ	depreciate rate	0.06
c_{min}	consumption floor	10,000
ρ_y	autocorrelation income	0.95
$\sigma_{y,\epsilon}^2$	variance innovation income (less HS, HS, College+)	(.033,.025,.016)
ρ_o	autocorrelation out-of-pocket	0.901
$\sigma_{o,\epsilon}^2$	variance innovation out-of-pocket (less HS, HS, College+)	(.175,.156,.153)

Table A.1: **Baseline Parameter Values**