Risk Diversification

Lecture 10

Putting all of your eggs in one basket and black swans

Financial Crises

Risk, uncertainty, and black swans

Though a helpful tool for understanding risk, **probability theory has its limitations**, especially in financial markets.

- Applying probability to financial markets requires that the **distribution of returns is well understood**.
- Returns modeled after historical data *may* be adequate predictors of returns **under ordinary circumstances only**; financial markets sometimes behave in a way that could not be predicted based on historical returns.
- Rare, high-impact events that cannot be predicted (but are often rationalized in hindsight) are known as **black swans**.
- A **severe stock market crash** is an example of a black swan that we consider next…

Stock market crashes

Long-term investors may witness considerable financial risk over their investment horizon. Occasionally, stock markets **crash**, and investors incur high losses over a period.

- Consider an investor who plans to retire in 35 years.
- The investor invests $10,000 each year in the stock market.
- The market returns 13% per year in all but one year.
- In that year, the stock market crashes and the investor incurs 47% loss. (The frequency of crashes is impossible to predict, but such returns might be witnessed once every 50-100 years, based on historical experience).

Do you think it would be worse for the investor if:

- The market crashes when he first starts saving
- The market crashes when he's about to retire

Market crashes and investment horizon

Under each scenario, the investors ending wealth and average return can be computed:

Crises at retirement:

Ending wealth: $2,897,408
Average return: 9.9%

Crises at beginning of investment horizon:

Ending wealth: $5,794,828
Average return: 12.7%

A stock market crash at retirement has a devastating effect on final wealth.

Market crash at retirement

The $10,000 contributions earn 13% per year for the first 34 years, and the balance is $5,456,808 at the end of 34th year.

Time Value of Money	
Mode	BEG
P/Y	1
PV	$0
PMT	-$10,000
N	34
I/Y	13%
FV=	$5,456,808

In the 35th year, another $10,000 contribution is made and the portfolio experiences a 47% loss, bringing the balance to $2,897,408.

Time Value of Money	
Mode	BEG
P/Y	1
PV	-$5,456,808
PMT	-$10,000
N	1
I/Y	-47%
FV=	$2,897,408

This is equivalent to a 9.88% average annual return over the 35 years.

Time Value of Money	
Mode	BEG
P/Y	1
PV	$0
PMT	-$10,000
FV	$2,897,408
N	35
I/Y=	9.88%

Market crash in first year

The first $10,000 contribution loses 47% of its value in the first year.

Time Value of Money

Mode	BEG
P/Y	1
PV	$0
PMT	-$10,000.00
N	1
I/Y	-47%

FV=	$5,300

Time Value of Money

Mode	BEG
P/Y	1
PV	-$5,300
PMT	-$10,000
N	34
I/Y	13%

FV=	$5,794,828

Over the next 34 years, $10,000 contributions are made and the investments return 13% per year. The ending balance is $5,794,828.

This is equivalent to a 12.74% average annual return over the 35 years.

Time Value of Money

Mode	BEG
P/Y	1
PV	$0
PMT	-$10,000.00
FV	$5,794,828
N	35

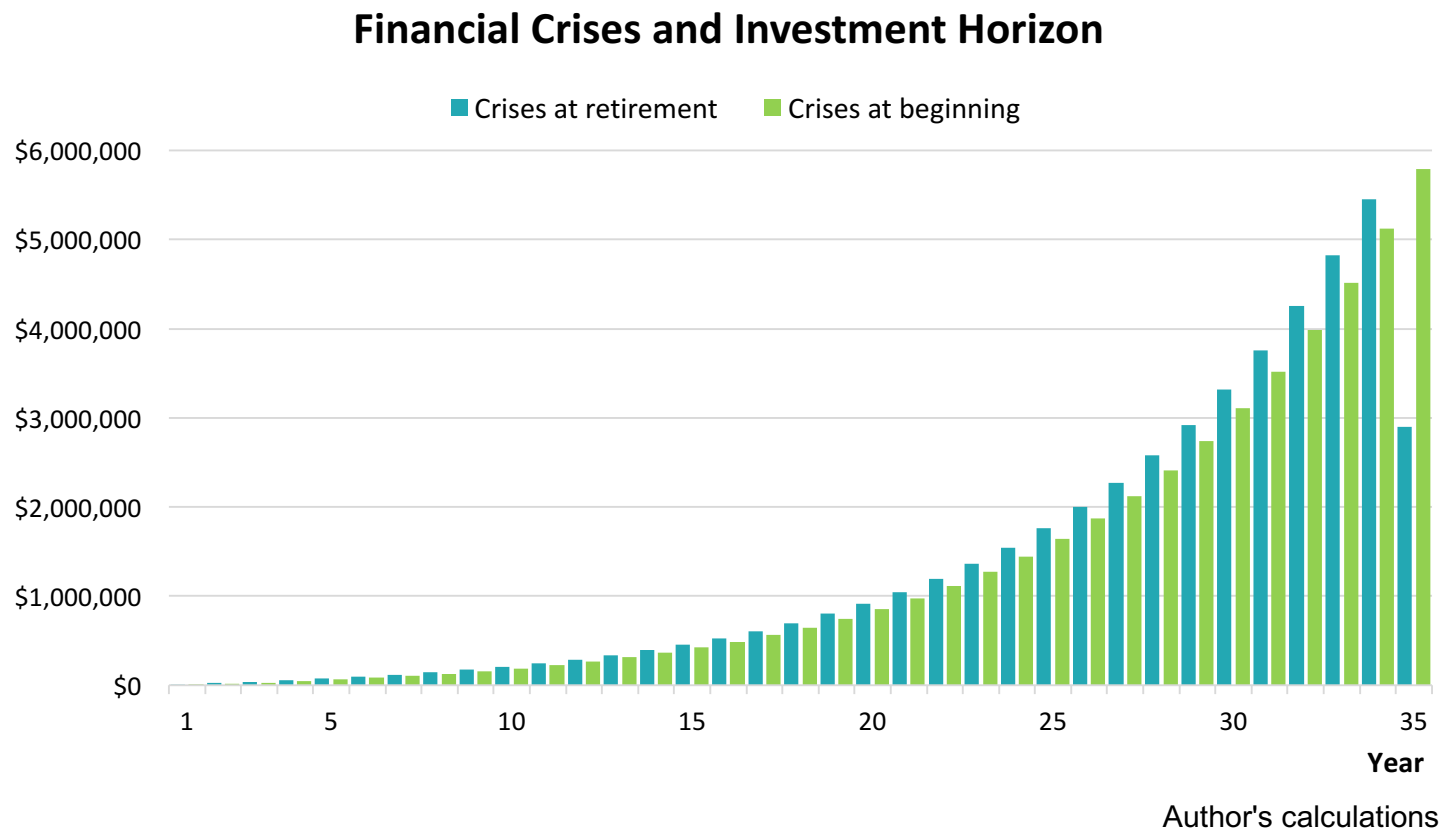| I/Y=	12.74%

Market crashes and investment horizon

When contributions are made periodically, the **order of returns matters**. Why is this the case?

- If the stock market crashes in the first year, only the investor's initial $10,000 contribution is subjected to the crash.

- If the stock market crashes in the final year, the investors total accumulated wealth – which includes all 35 $10,000 contributions – is subjected to the stock market crash.

Market crashes and investment horizon

The following graph demonstrates growth of wealth under the two scenarios over time:

Financial Crises and Investment Horizon

■ Crises at retirement   ■ Crises at beginning

Year

Author's calculations

(Note the collapse in wealth when the crises occurs at retirement.)

Market crashes and investment horizon

This example has **real-world implications**:

- If a crises occurs at the end of an investor's investment horizon, the investor's **entire accumulated wealth is affected**. But if the crises occurs at the beginning, only the investor's initial contributions are affected.
- Thus, when the investment horizon becomes short as an investor approaches retirement, the short-term volatility of the stock market threatens the investor's entire accumulated wealth.
- However, because the long-run expected return of the market is high and the volatility is averaged over long periods, it is sensible to invest in stocks when the horizon is long.
- Therefore, an investor would be well-advised to invest in stocks when young, but to transition out from stocks and into less risky assets, such as Treasuries, as retirement approaches.

Risk Diversification

Knowledge of risk diversification

To test knowledge of risk diversification, we asked:

Please tell me whether this statement is true or false. "Buying a single company's stock usually provide a safer return than a stock mutual fund."

a) True
b) False
c) Don't know
d) Prefer not to say

Knowledge of risk diversification

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a) True

b) False

c) Don't know

d) Prefer not to say

46% answered correctly

44% answered "Don't know"

Source: 2015 National Financial Capability Study

Independence

Imagine the following deal:

- I flip a fair coin.
- If it comes up heads, I give you a $1.
- If it comes up tails, you give me a $1.

How likely are you to lose a dollar?

- You have a 50% chance of losing a dollar.
- And you also have a 50% chance of gaining a dollar.
- If we play this game once, there is no other outcome.

Independence

Now, consider a different deal:

- I flip a fair coin **20 times**.
- For every heads, I give you $0.05.
- For every tails, you give me $0.05.

Could you still lose a dollar?

- Yes, if I flip 20 straight tails, you would owe me $1.00.
- And if I flip 20 straight heads, I would owe you a dollar.
- But there are more outcomes. If I flip 10 tails and 10 heads, for example, we would owe each other nothing, on net.

How likely are you to lose a dollar?

- Very unlikely. It would be very rare to flip 20 straight tails.
- In fact, this will only happen 1 in 1,000,000 times!

Independence

The reason it is very unlikely to flip 20 straight heads is because coin flips are **independent**.

- The outcome of one coin flip does not influence the next.
- If one coin flip is tails, the next is no more likely to be tails.
- If we flip 20 coins, we are almost guaranteed to have some heads and some tails.
- The heads and tails will cancel each other out in our game, so it is unlikely that either of will lose very much.
- Instead, the outcome is likely to be around the average outcome: we owe each other nothing.
- For example, the chance that either of us owe the other $0.30 or more is less about 10%.
- And if we flipped 100 coins for a penny apiece, that chance would be less than 1%!

Risky assets

Risky assets behave in a similar way. There returns are random and we can learn something from comparing them to a coin flip.

The following chart shows the returns on an asset that returns 2% half the time and 12% the other half of the time. On average, such an asset will return 7%, but it is risky.

Return Dispersion

|														
0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%
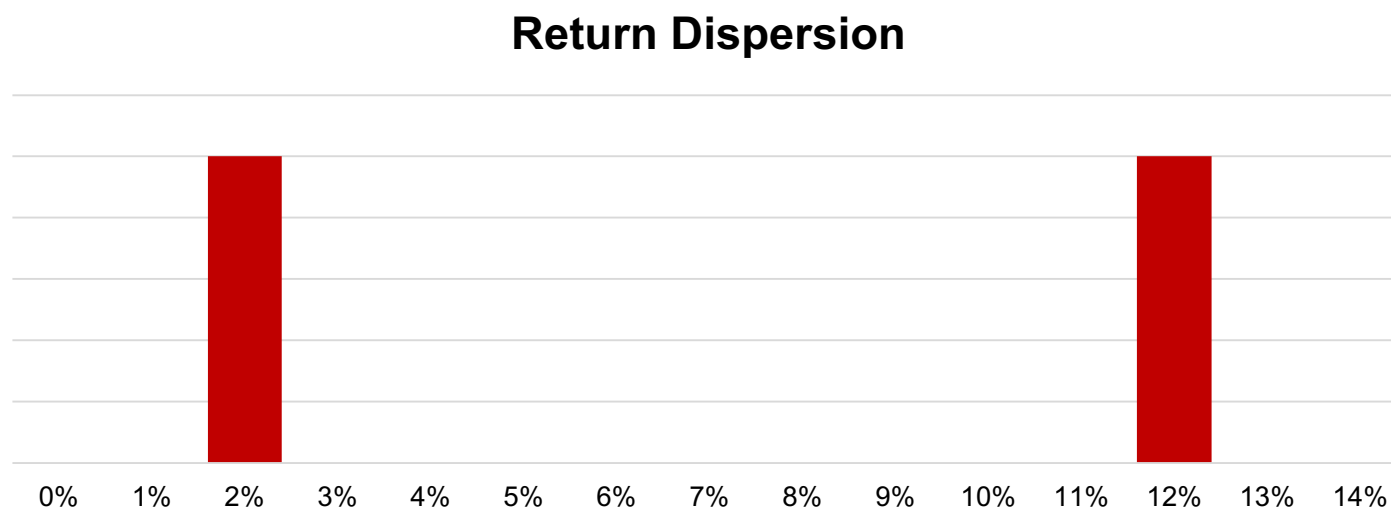
Two risky assets

If two risky assets are **independent**, it is less likely that both of them will do badly at once.

The following chart compares a portfolio invested in the one risky asset described earlier to a portfolio invested equally in two similarly behaving assets. Each asset will perform poorly 50% of the time, but they will only do so at the same time 25% of the time. **Investing in two assets, rather than one, decreases the risk.**

Two Assets and Return Dispersion

■ One Asset   ■ Two Assets
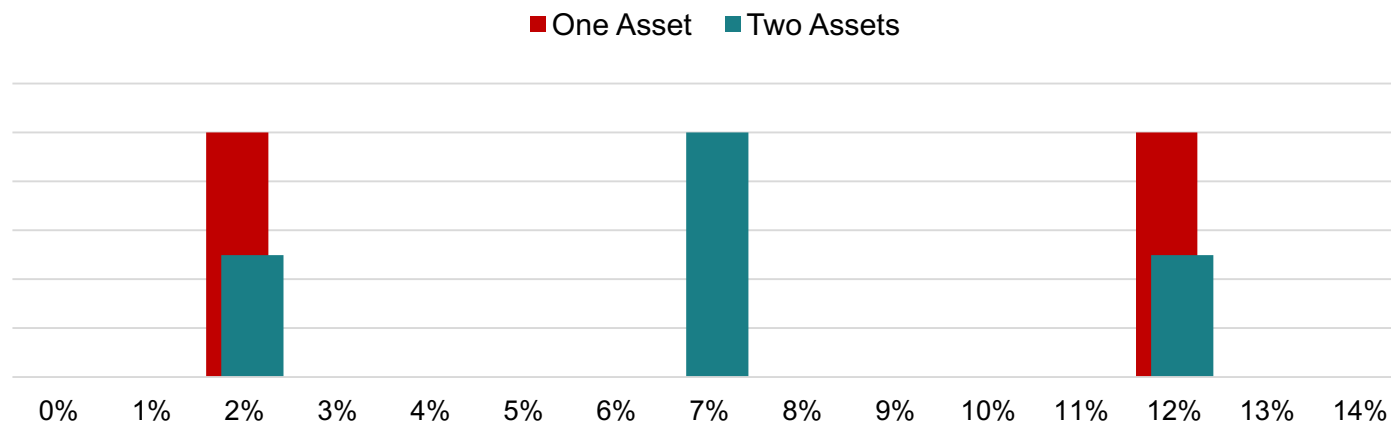
Two risky assets

To see that this is the case, we can model this scenario by imagining two coin flips, each of which determines the return on one of the investments:

- If the first coin lands heads up, the return on the first investment is 12%. If it lands tails up, the return on the first investment is 2%.
- Similarly, the return on the second investment is 12% if the second coin flip is heads but only 2% if it is a tails.

The outcomes, and their probabilities, are listed in the following table:

| First Flip	Second Flip	Probability	Return
Heads	Heads	0.25	12%
Heads	Tails	0.25	7%
Tails	Heads	0.25	7%
Tails	Tials	0.25	2%

Spreading your investment between two assets decreases the probability of a low return from 50% to 25%.

Multiple risky assets

And spreading your portfolio across more assets further decreases the risk of a low return.

When several such assets are considered, it becomes less and less likely that they all perform poorly at once. In a portfolio spread equally across a number of such assets, the over-performing assets will cancel out the underperforming assets, and it **becomes less and less likely that the portfolio's return differs from the average**.

Multiple Assets and Return Dispersion
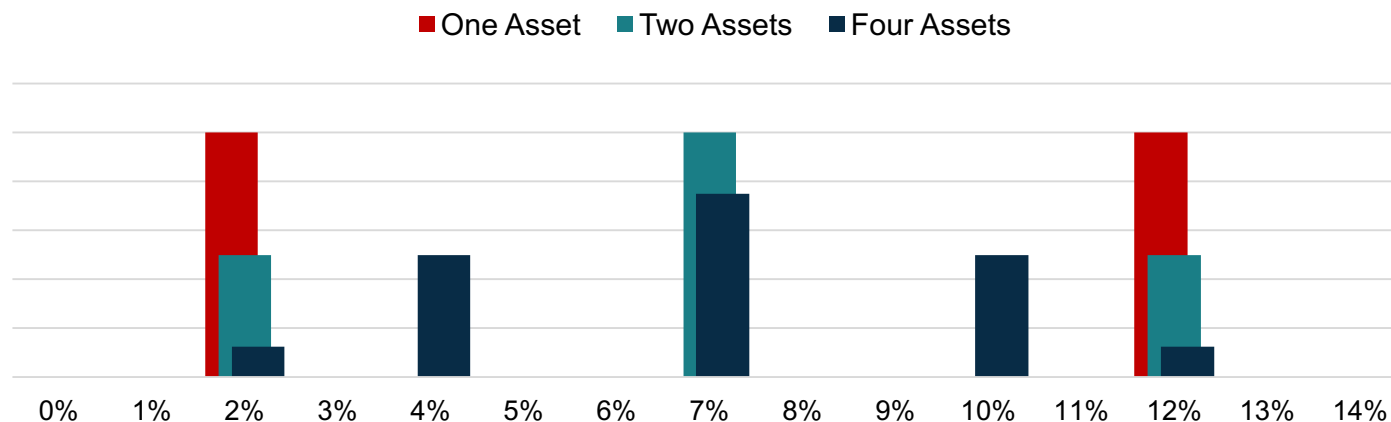
Risk diversification

This phenomenon is known as **risk diversification**.

- Spreading an investment across multiple assets is known as **diversification**.
- The **principle of diversification** states that spreading an investment across multiple assets will eliminate some of the risk.
- It's less likely that all assets will perform poorly at the same time. When one performs poorly, another may perform well, and the two will cancel out.
- This is the financial equivalent of "**don't put all of your eggs in one basket**".
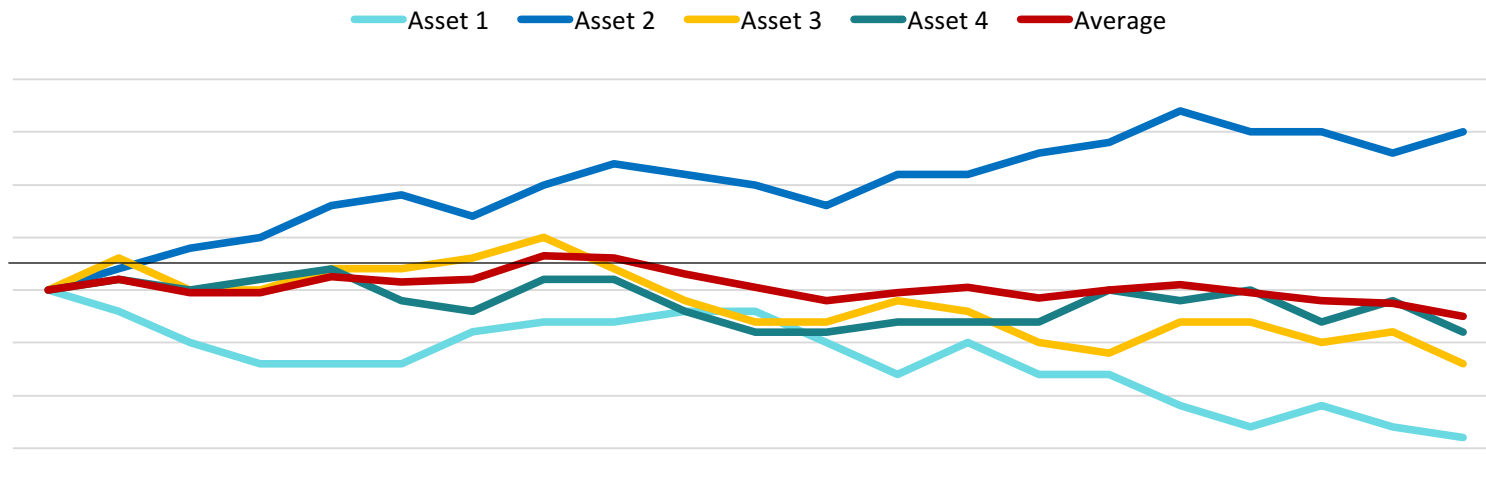
Risk diversification

The graph below compares the path of the prices of four assets to the average of all four prices.

- The individual assets are more volatile than the average.
- This is because, when some assets move up, others move down, and the average is smoothed out.
- This is once again the **law of large numbers** at work: the average of many random variables is less likely to deviate far from its expectation.

Risk Diversification and Volatility

Asset 1 — Asset 2 — Asset 3 — Asset 4 — Average

Risk diversification and standard deviations

Mathematically, diversifying across multiple assets reduces the standard deviation of returns.

When invested in the **single asset** described above, we saw that the expected return was 7%. The **standard deviation is 5%.**
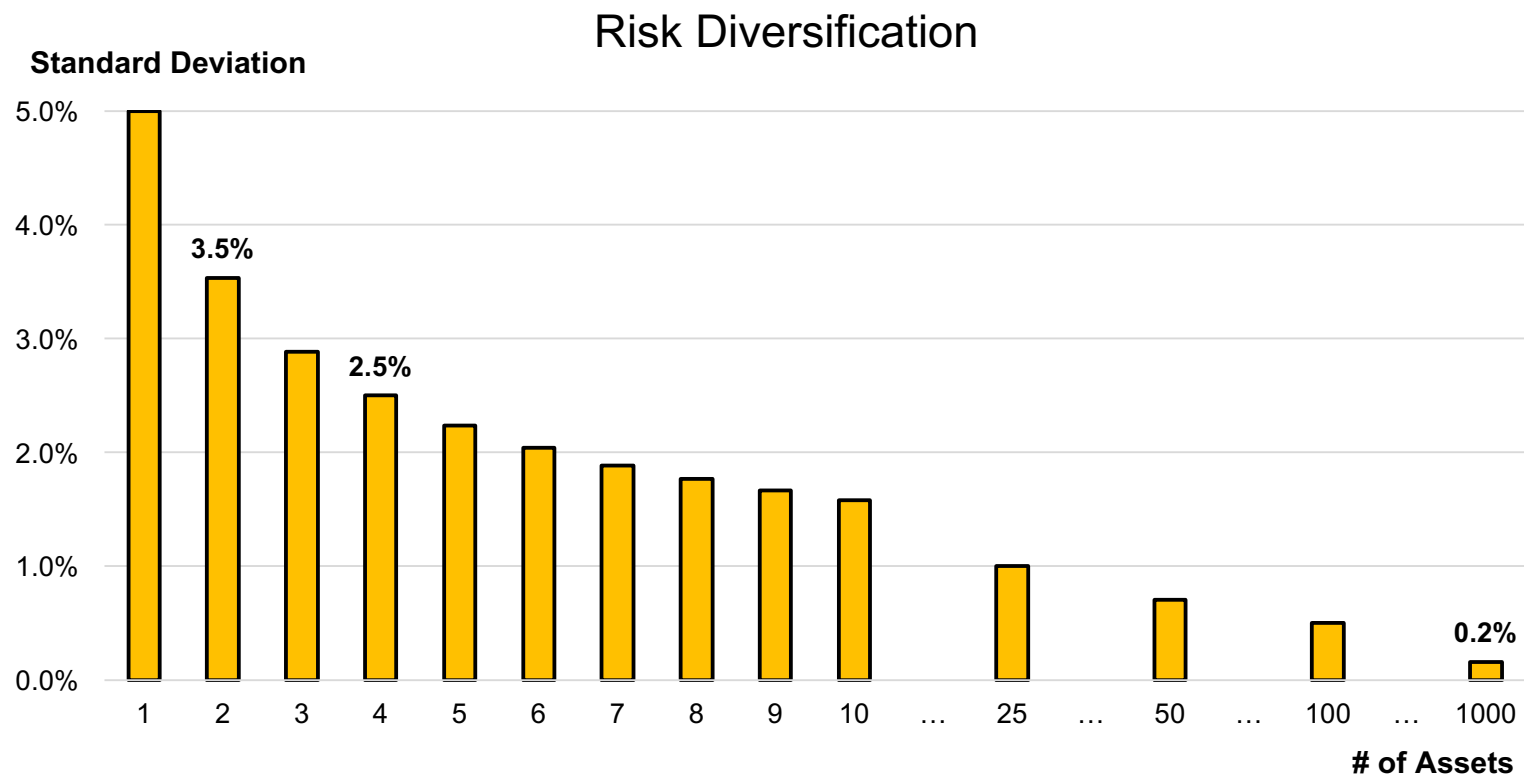
With **two assets** the expected return is still 7%, but the **standard deviation drops to 3.5%.**

With **three assets** the **standard deviation drops even further to 2.5%.**

Risk diversification and standard deviations

And eventually, as the number of assets continues to increase, the standard deviation will converge to zero…

Limits to Diversification

Stock diversification

We have only considered the effects of diversification on a *hypothetical* investment. Let's now consider the effects of diversification on a stock portfolio

- Historically, average return on a large company stock has been around 13%.
- The standard deviation of the returns on a **single** large company stock has been 50%. This is a lot of risk. It means losses around 40% are not unlikely.
- In a portfolio of 500 large company stocks, the average return is still 13%, but the standard deviation is around 20%. This makes losses of 40% highly unlikely (but not impossible).
- This decreased risk is a side effect of diversification: **by investing in a portfolio of stocks, rather than a single stock, risk is reduced.**
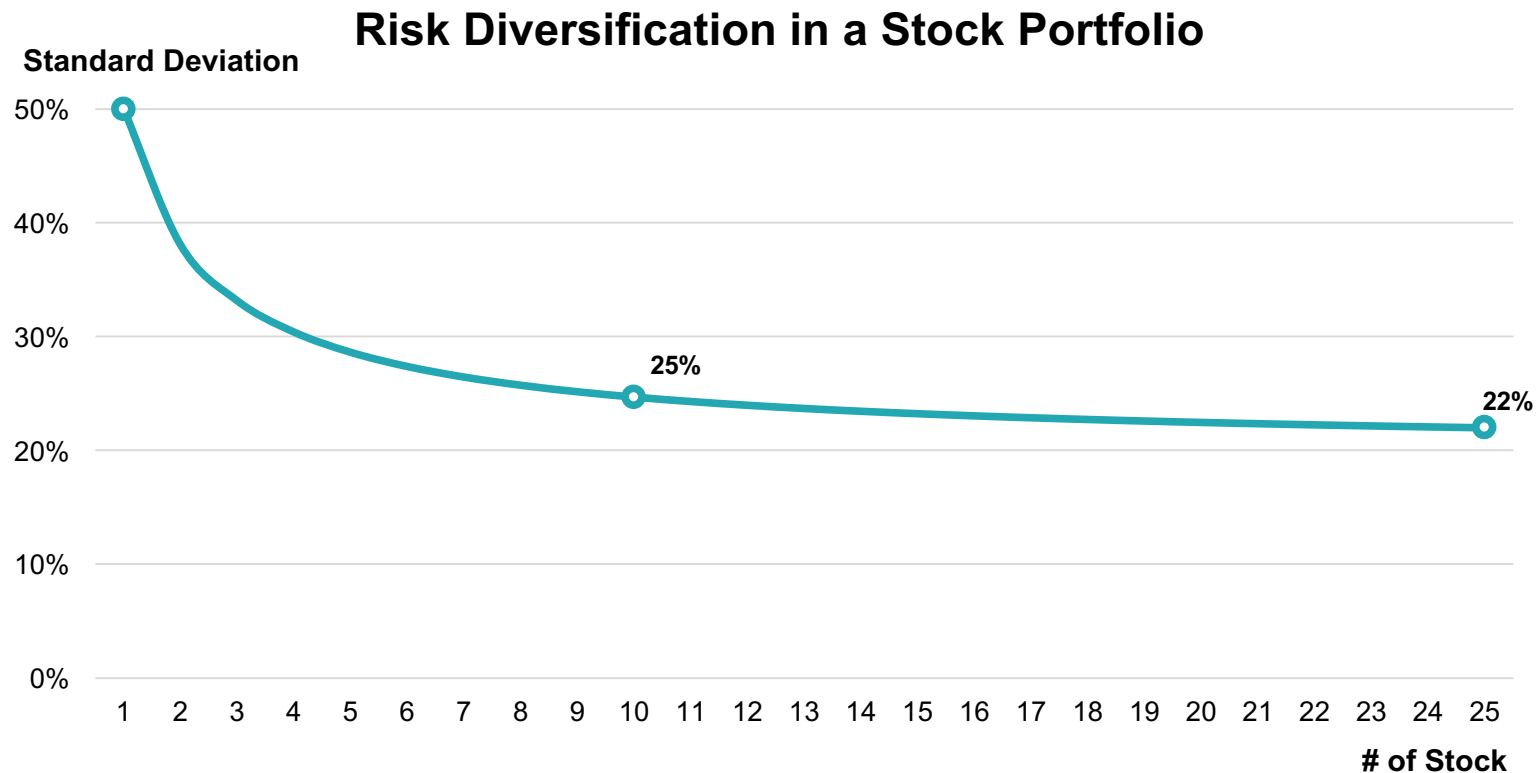
Correlation

For actual financial asset classes, like stocks, the effect of diversification is limited by correlation between assets.

- Risk diversification depends on outcomes in one direction being offset by outcomes in the opposite direction across a large sample of outcomes.
- However, if the outcomes are **not independent** across trials such that if one outcome is in one direction, the other outcomes (though still random) are more likely to be in that same direction, the effects of diversification are limited.
- When two variables tend to move in the same direction they are said to be **positively correlated**.
- For example, although company-specific factors affect a stock's price, general economic conditions affect all companies. This creates a **positive correlation across stocks that limits the effects of diversification**.
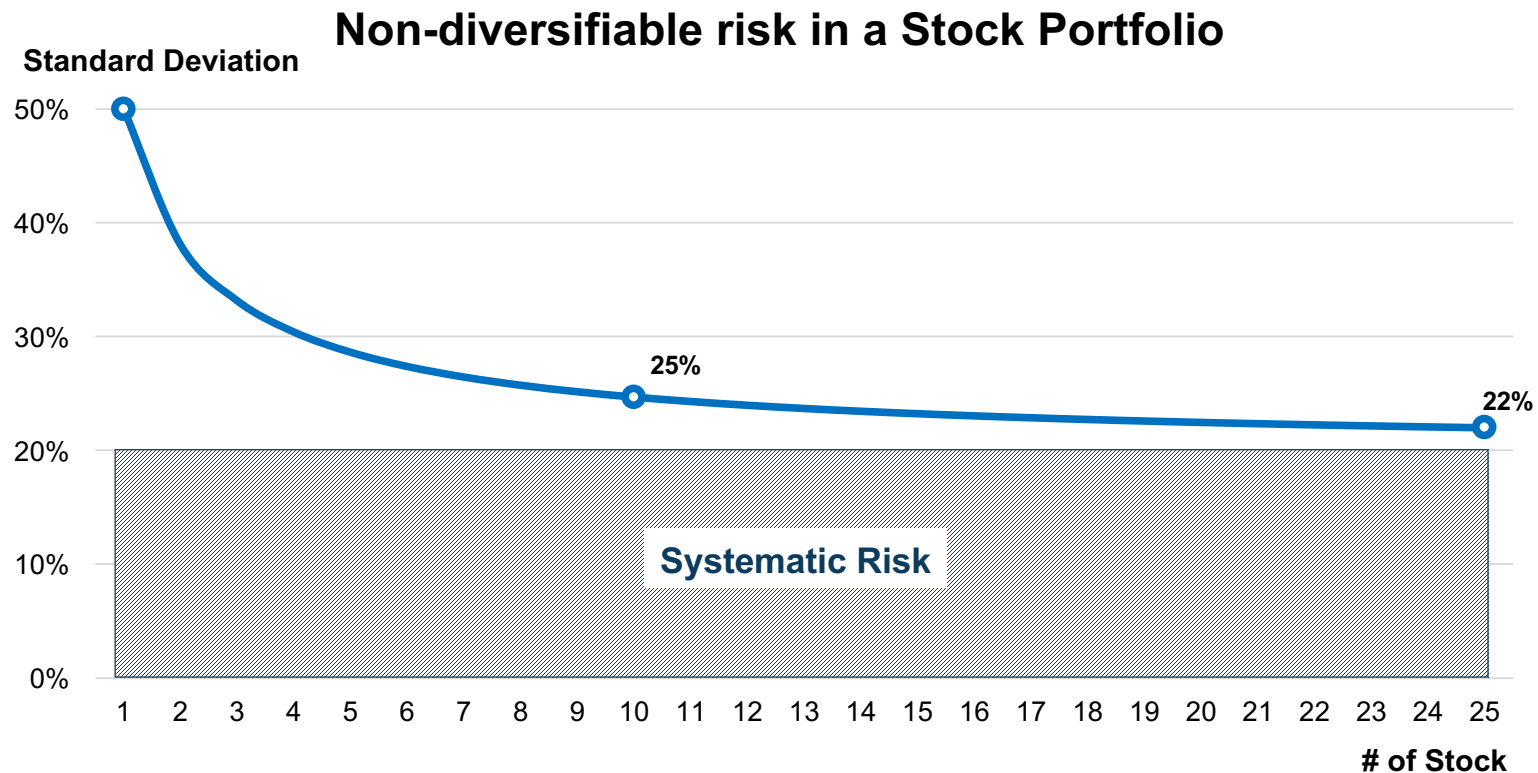
Limits to stock diversification

The following chart shows the effect on the standard deviation of a stock portfolio as the number of stock in the portfolio increases:

Risk Diversification in a Stock Portfolio

Limits to stock diversification

The standard deviation of the portfolio converges to 20%. In fact, this remaining risk cannot be diversified away and is known as **non-diversifiable risk** or **systematic risk**.

Non-diversifiable risk in a Stock Portfolio

Standard Deviation

50%

40%

30%

25%

22%

20%

10%

Systematic Risk

0%

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

# of Stock

The correlation coefficient

The degree to which two assets are **correlated** may be measured with the **correlation coefficient**.

Though it's not necessary for this course, for reference, the correlation coefficient between two random variables X and Y is calculated as:

$$\rho_{X,Y} = \frac{E[(X - \mu_x)(Y - \mu_Y)]}{\sigma_x \sigma_Y}$$

Where μ_x is the expected value of X and σ_x is the standard deviation of X.

The correlation coefficient

The correlation coefficient may be between -1 and 1.

- A correlation coefficient between 0 and 1 indicates that the two variables are **positively correlated**: when one is high, the other also tends to be high.
- A correlation coefficient between -1 and 0 indicates that two variables are **negatively correlated**: when one is high, the other tends to be low.
- A correlation coefficient of 0 indicates that two variables are **independent**: the outcome of one has no effect on the other.
- A coefficient of exactly 1 (or -1) indicates that two variables are **perfectly correlated**, and that the two variables *always* move in the same (or opposite) directions.

Diversification Across Asset Classes

Diversification

Video about diversification

"How Diversification Decreases Risk"

http://gflec.org/education/educational-videos/

Diversification across asset classes

The correlation between investments of the same type will limit the effects of diversification across that asset class. But further benefits may be obtained by diversifying across different asset classes.

- The correlation between one randomly selected stock is high enough to limit the effect of diversification.

- However, further diversification benefits may be realized by **diversifying across asset classes**.

- For example, although the returns to stocks and bonds may be still correlated, dividing an investment across the two asset classes provides additional diversification benefits.

- To fully diversify your portfolio **invest in many asset classes**: large company stocks, small company stocks, international stocks, corporate bonds, US Treasuries, sovereign bonds, commodities, and even real estate.

Applications of Risk Diversification

Employee stock plans

Workers with a large portion of their portfolio in their company's stock may be subjecting themselves to unnecessary risk.

Ex. A worker has $100,000 worth of savings, $50,000 of which are held in the company he works for. The remaining $50,000 is invested in a well diversified portfolio of large company stock.

Assuming his company is a typical large company, the expected return on both his company stock and his portfolio should be around 13%. The standard deviation on his employer's stock, however, should be around 50%, while the standard deviation on his diversified portfolio will be around 20%.

The standard deviation on his total portfolio will be around 30%. By selling his company stock and investing the proceeds in a diversified portfolio, he may reduce risk without effecting his expected return.

Further, it is especially risky for his savings to be correlated with his source of income. If his company fails, he may lose his savings at the same time he loses his income (think about the employees at Enron and Lehman Brothers).

Diversification in business

The principle of risk diversification extends beyond personal investing. It is highly relevant to business as well.

- Entrepreneurs bear a lot of risk from a lack of diversification. All of an entrepreneur's wealth is often tied to the single investment in their small business, and this income source may become highly volatile.

- To diversify their income, large businesses often invest in multiple product lines or even business lines. Companies that do so broadly are known as **conglomerates**.

- However, if corporations or businessmen diversify too widely, they may not be able to concentrate on their "core competencies", and their business may suffer. As the steel tycoon, Andrew Carnegie, famously said…
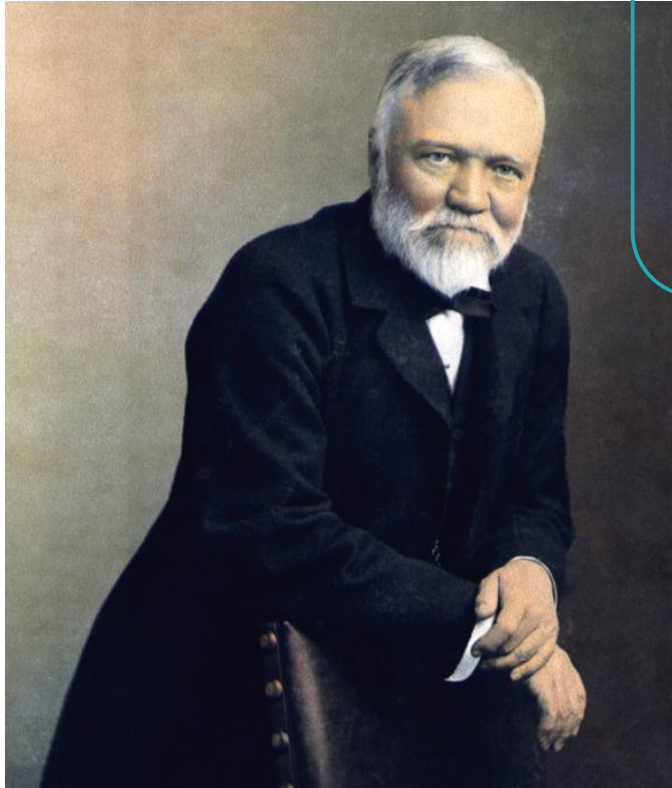
Diversification in the insurance industry

Insurance is another industry that benefits from risk diversification. In fact, the business model depends on it!

- An insurance companies collects premiums from a policyholders and pays them if a certain event occurs.
- For example, an auto insurance company pays for vehicle repairs if a policyholder gets into an accident.
- If all automobile accidents were perfectly correlated – that is, if one policyholder gets into an accident means all policyholders do – the insurer would have to pay everybody at once.
- To have the money to do so, the insurer would need to charge everybody a premium equal to the cost of repairing an automobile.
- But who would pay those premiums? You might as well just pay for the auto repairs yourself!
- The industry depends on some independence between car accidents. Although there's a chance any given driver will get into an accident, not every driver necessarily will!

Most wealth in homes

Many consumers don't diversify their wealth. Instead, they hold most of it in their homes.

Diversification in real estate

Risk diversification is also important in real estate.

- For many consumers, their home is their most valuable assets; most of their wealth is in their home.

- They are not well diversified! Their wealth is greatly influence by the performance of their home's price.

- Similarly, because property is so expensive, it is hard for real estate investors to diversify across multiple properties. To invest in ten $500,000 houses would require an investment of $5,000,000 (or $1,000,000 with 20% down mortgages).

- If a real estate investor can only afford to invest in one property, their portfolio performance will be tied to one asset and may be highly volatile.

- **Real estate investment trusts (REITs)**, offer a solution to real estate investors. REITs invest in properties across multiple geographies and of multiples types. This offers investors access to a diversified portfolio of real estate at an affordable price. (Of course, there is still **systematic risk**.)

Today we learned…

- ✓ Risk and dispersion
- ✓ Risk on stocks vs. bonds
- ✓ Financial crises
- ✓ Risk diversification
- ✓ Limits to diversification
- ✓ Diversification across asset classes
- ✓ Applications of risk diversification