

## Practice Quiz 7: Mortgage Borrowing (Solutions)

1. Jack and Jill are looking for a new home and find a \$400,000 house that they like. Their bank offers them a 30-year mortgage at a 7% APR with a 15% down payment and no points. Calculate the monthly payment for this mortgage.

**Ans.** With a down payment of 15%, Jack and Jill will need to finance  $0.85 * \$400,000 = \$340,000$  of the house price. The monthly payment can be found using a financial calculator (the monthly interest rate is  $7\%/12 = 0.583\%$ ):

<u>Time Value of Money</u>	
<b>PV</b>	\$340,000
<b>FV</b>	\$0
<b>i</b>	0.583%
<b>n</b>	360
<b>PMT=</b>	-\$2,262.03

Thus, the monthly payment will be \$2,262.03.

2. Their bank also offers Jack and Jill a 15-year mortgage at a 6.25% APR for the \$400,000 house, also with a 15% down payment and no points. Calculate the payment on this mortgage and compare the total interest cost of this mortgage to the total interest cost of the 30-year mortgage in problem 1.

**Ans.** Using a financial calculator (with a monthly interest rate of  $6.25\%/12 = 0.521\%$ ):

<u>Time Value of Money</u>	
<b>PV</b>	\$340,000
<b>FV</b>	\$0
<b>i</b>	0.521%
<b>n</b>	180
<b>PMT=</b>	-\$2,915.24

The monthly payment on the 15-year mortgage will be \$2,915.24.

3. If Jack and Jill are willing to pay two points, their bank will drop the APR on the 30-year mortgage in problem 1 to 6.5%. Calculate the implicit APR on this mortgage assuming Jack and Jill stay in the home for (a) 30 years, (b) 10 years, and (c) 4 years.

**Ans.** With points, the APR on the 30-year mortgage will drop to 6.5% and the monthly payment will be (the monthly interest rate is  $6.5\%/12 = 0.542\%$ ):

<u>Time Value of Money</u>	
<b>PV</b>	\$340,000
<b>FV</b>	\$0
<b>i</b>	0.542%
<b>n</b>	360
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<b>PMT=</b>	-\$2,149.03

Two points will cost  $0.02 * \$340,000 = \$6,800$  up front.

(a) If Jack and Jill remain in the home for 30 years, the cash flows associated with their mortgage will be: A \$340,000 receipt from their loan and a \$6,800 payment for the points, for a net cash flow of \$333,200 today. 360 monthly payments of \$2,149.03. The implicit monthly interest rate on such a series of payments is:

<u>Time Value of Money</u>	
<b>PV</b>	\$333,200
<b>FV</b>	\$0
<b>PMT</b>	-\$2,149.03
<b>n</b>	360
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<b>i=</b>	0.558%

This corresponds to an APR of  $12 * 0.558\% = 6.7\%$ . Because this is less than the 7.0% APR with no points, it will be financially cheaper to pay points if Jack and Jill plan to stay in the home for 30 years.

(b) If Jack and Jill remain in the home for 10 years, they will have the following mortgage balance to repay after selling the home:

<u>Time Value of Money</u>	
<b>PV</b>	\$340,000
<b>PMT</b>	-\$2,149.03
<b>i</b>	0.542%
<b>n</b>	120
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<b>FV=</b>	-\$288,239.05

In this case, the cash flows for the loan will be: \$333,200 in net proceeds today, 120 monthly payments of \$2,149.03, and a final payment of \$288,239.05 to close out their debt after 10 years. The implicit monthly interest rate on such a debt is:

<u>Time Value of Money</u>	
<b>PV</b>	\$333,200
<b>FV</b>	-\$288,239.05
<b>PMT</b>	-\$2,149.03
<b>n</b>	120
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<b>i=</b>	0.566%

This corresponds to an APR of  $12 \times 0.566\% = 6.8\%$ . Because this is less than the 7% APR without points, paying the points will be cheaper for Jack and Jill if they stay in the home for 10 years.

(c) If Jack and Jill remain in the home for 4 years, they will have the following mortgage balance to repay after selling the home:

<u>Time Value of Money</u>	
<b>PV</b>	\$340,000
<b>PMT</b>	-\$2,149.03
<b>i</b>	0.542%
<b>n</b>	48
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<b>FV=</b>	-\$323,202.62

In this case, the cash flows for the loan will be: \$333,200 in net proceeds today, 48 monthly payments of \$2,149.03, and a final payment of \$323,202.62 to close out their debt after 4 years. The implicit monthly interest rate on such a debt is:

<u>Time Value of Money</u>	
<b>PV</b>	\$333,200
<b>FV</b>	-\$323,202.62
<b>PMT</b>	-\$2,149.03
<b>n</b>	48
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<b>i=</b>	0.591%

This corresponds to an APR of  $12 \times 0.591\% = 7.1\%$ . Because this is greater than the 7% APR without points, paying points will be more expensive for Jack and Jill if they only stay in the home for 4 years.

4. If a student graduates with a \$12,000 in student debt at a 5% APR to be repaid over 10 years, what will her monthly payment be?

**Ans.** The monthly payment can be found using a financial calculator ( $\text{APR}/12 = 5\%/12 = 0.417\%$ ):

<u>Time Value of Money</u>	
<b>PV</b>	\$12,000
<b>FV</b>	\$0
<b>i</b>	0.417%
<b>n</b>	120
<b>PMT=</b>	-\$127.28

The monthly payments will then be \$127.28.

5. Albert just graduated from a top law school and was hired by a successful practice in his home city with a starting annual salary of \$120,000, but carries \$30,000 in student debt at a 6% APR to be repaid over 10 years. Albert has no other debt. Property tax in his city is 2% and homeowner's insurance is 0.5% of a home's value. With a down payment of 20%, his bank will provide him a 30-year mortgage at a 4.5% APR. If his bank requires a housing expense ratio of 28% and a maximum total expense ratio of 35%. What is the most expensive house Albert can finance?

**Ans.** The housing expense ratio of 28% requires that:

$$\text{Payment} + \text{Taxes} + \text{Insurance} \leq 0.28 * \text{Monthly Income}$$

Albert has a monthly income of  $\$120,000/12 = \$10,000$  per month. The monthly tax and insurance payments, in terms of the house price, can be found to be:

$$\begin{aligned} \text{Taxes} &= \frac{0.02}{12} * \text{House Price} = .00167 * \text{House Price} \\ \text{Insurance} &= \frac{0.005}{12} * \text{House Price} = .00042 * \text{House Price} \end{aligned}$$

Using the fixed payment formula, the monthly mortgage payment can be found in terms of the house price:

$$\frac{0.80 * \text{House Price}}{\left( \frac{1}{(0.045/12)} - \frac{1}{(0.045/12)(1 + 0.045/12)^{360}} \right)} = .00405 * \text{House Price}$$

Plugging all of this into the housing expense ratio:

$$(0.00405 + 0.00167 + 0.00042) * \text{House Price} \leq 0.28 * \$10,000$$

$$\text{House Price} \leq \frac{.28 * \$10,000}{0.00405 + 0.00167 + 0.00042} = \$456,026$$

The housing expense ratio thus requires that Albert finance a house worth no more than about \$456,000.

For the total expense ratio of 35%, debt payments must also be considered:

$$\text{Payment} + \text{Taxes} + \text{Insurance} + \text{Debt Payments} \leq 0.35 * \text{Monthly Income}$$

Albert's monthly student debt payment can be found using a financial calculator:

<u>Time Value of Money</u>	
<b>PV</b>	\$30,000
<b>FV</b>	\$0
<b>i</b>	0.5%
<b>n</b>	120
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<b>PMT=</b>	-\$333.06

All other payments remain the same and the constraint for the total expense ratio is:

$$(0.00405 + 0.00167 + 0.00042) * \text{House Price} - \$777.14 \leq 0.35 * \$10,000$$

$$\text{House Price} \leq \frac{.35 * \$10,000 - \$333.06}{0.00405 + 0.00167 + 0.00042} = \$515,788$$

In this case, the housing expense ratio is more binding and Albert can finance a house worth up to \$456,000 under these conditions.