

## Practice Quiz 9: Basic Probability and Risk vs. Return (Solutions)

1. What is the probability that, when two dice are rolled, the sum of the numbers on the two dice is five?

**Ans.** There are  $6 \times 6 = 36$  possible outcomes from rolling two dice. Of these 36 outcomes, 4 are such that the sum of the numbers on the two dice is five: (1,4), (2,3), (3,2), and (4,1). Therefore, the probability that the sum of the numbers on the two dice is five is  $4/36 = 11.1\%$ .

2. A casino offers a bet in which a gambler flips a coin three times. The gambler wins \$3 for flipping three heads, \$1 for flipping two heads, and nothing otherwise. What is the expected value of this game? Will the gambler make money, on average and in the long-run, if this game costs \$0.50 to play? What if the game costs \$1?

**Ans.** The possible outcomes from flipping a coin three times are {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. The probability of three heads is therefore  $1/8$  and the probability of two heads is  $3/8$ . The expected value of this bet is:

$$\frac{1}{8} * \$3 + \frac{3}{8} * \$1 = \$0.75$$

If the game costs \$0.50 to play, then on average, the gambler will profit \$0.25 per play. But if the game costs \$1.00 to play, then the gambler will lose \$0.25 per game, on average.

3. An investment returns 0% a quarter of the time, 16% another quarter of the time, and 8% the remaining half of the time. Calculate the expected return on this investment.

**Ans.** The expected return on this investment is:

$$\frac{1}{4} * 0\% + \frac{1}{4} * 16\% + \frac{1}{2} * 8\% = 8\%$$

4. Discuss the law of large numbers.

**Ans.** The law of large numbers states that, with a large number of trials, the proportion of time that an even occurs will converge towards its probability of occurring. Specifically, as the number of trials increases, it becomes increasingly less likely that the observed proportion of occurrences of an event will differ materially from its probability of occurring. As a corollary, the average value of an event is unlikely to differ materially from its expected value over a large number of trials.

**5.** Discuss the standard deviation and how it relates to financial risk.

**Ans.** The standard deviation is a mathematical measure of the dispersion of a random variable. The higher a random variable's standard deviation, the more widely dispersed are the potential outcomes. Intuitively, a wider range of random returns for a financial investment is considered to be more risky, and so the standard deviation of returns on an investment is often used as a measure of its risk. For example, because stock returns have been historically more volatile than bond returns, they are seen as more risky than bonds, and the higher standard deviation on stock returns is consistent with this intuition.

**6.** Discuss the relationship between risk and return.

**Ans.** There is a tradeoff between return and risk. Assets that generate higher average returns often entail more risk. For example, assets that have historically generated higher returns, such as stock, have also exhibited more volatile returns historically.

To understand why this is the case, imagine that there were two assets that offered the same expected return, but one was riskier than the other. If this were the case, no rational investor would want to hold the riskier asset. Therefore, all investors would purchase the less risky asset and sell the riskier asset and, by supply and demand, the price of the less risky asset would be bid upward and the price of the risky asset would fall. Because returns are inversely related to prices, this implies that the expected return on the less risky asset would fall while the return on the riskier asset increases. This would continue until the return on the riskier asset offered a sufficient premium to compensate investors for the increased risk.

7. An investor uses a discount rate of 6% to value riskless investments. The investor is considering an investment that returns either \$30 or \$70, each with 50% probability, each year for four years. The cost of the investment is \$150. If the investor discounts the investment using the 6% discount rate, should she undertake the investment? If she is risk-averse, will she use a 6% discount rate? If she requires a risk premium of 8% on such a risky investment, should she undertake the investment?

**Ans.** The expected annual cash flow from this investment is:

$$0.5 * \$30 + 0.5 * \$70 = \$50$$

If the investor discounts the investment with a 6% discount rate, the net present value of the investment will be:

$$NPV = \frac{\$50}{1.06} + \frac{\$50}{1.06^2} + \frac{\$50}{1.06^3} + \frac{\$50}{1.06^4} - \$150 = \$23.26$$

And since the NPV is positive, she should undertake the investment if she requires a 6% return on such an investment. However, if she is risk-averse, she will not be satisfied with the same 6% return for riskless investments on this risky investment. If she requires an 8% risk premium on such a risky investment, she will require a 6% + 8% = 14% and value the project as:

$$NPV = \frac{\$50}{1.14} + \frac{\$50}{1.14^2} + \frac{\$50}{1.14^3} + \frac{\$50}{1.14^4} - \$150 = -\$4.31$$

Because the NPV is negative, the project is not expected to yield her required return of 14%, and so she should not undertake the project.

8. Matt and Lindsay are both currently 20 years old and plan to retire in 45 years at 65. They hope to withdraw \$60,000 from their savings at the *beginning* of each year to live off of in retirement and want to save enough to last 40 years. They estimate they can earn an average return of 8% on their investments while saving for retirement, but after that will shift into less risky assets which they expect should yield around 3% per year. How much must Matt and Lindsay set aside at the *beginning* of each year to meet their goals? If Matt and Lindsay set aside this amount at the end of each year and earn 8% as expected during the first 44 years, but experience a stock market crash and *lose* 40% during the 45<sup>th</sup> year, how much will they have in their account at retirement? How much may they withdraw at the beginning of each year for 40 years in retirement if they earn the expected 3% on their savings? (Assume no inflation.)

**Ans.** To withdraw \$60,000 at the beginning of each year for 40 years from an account earning 3% interest, the couple must accumulate \$1,428,493 by retirement:

| <u>Time Value of Money</u> |              |
|----------------------------|--------------|
| <b>BGN</b>                 | BGN          |
| <b>P/Y</b>                 | 1            |
| <b>FV</b>                  | \$0          |
| <b>PMT</b>                 | \$60,000     |
| <b>N</b>                   | 40           |
| <b>I/Y</b>                 | 3%           |
| <hr/>                      |              |
| <b>PV=</b>                 | -\$1,428,493 |

To do so in 45 years with an investment account that earns 8% per year, they must make annual contributions of \$3,422.15 at the beginning of each year:

| <u>Time Value of Money</u> |             |
|----------------------------|-------------|
| <b>BGN</b>                 | BGN         |
| <b>P/Y</b>                 | 1           |
| <b>PV</b>                  | \$0         |
| <b>FV</b>                  | \$1,428,493 |
| <b>N</b>                   | 45          |
| <b>I/Y</b>                 | 8%          |
| <hr/>                      |             |
| <b>PMT=</b>                | -\$3,422.15 |

If they do so for 44 years and earn 8% per year, they will accumulate \$1,319,258:

| <u>Time Value of Money</u> |             |
|----------------------------|-------------|
| <b>BGN</b>                 | BGN         |
| <b>P/Y</b>                 | 1           |
| <b>PV</b>                  | \$0         |
| <b>PMT</b>                 | -\$3,422.15 |
| <b>N</b>                   | 44          |
| <b>I/Y</b>                 | 8%          |
| <hr/>                      |             |
| <b>FV=</b>                 | \$1,319,258 |

At the beginning of the year, they must make their annual contribution of \$3,422.15, which brings their total balance to \$1,322,680. If this earns 8%, as planned, this will grow to their target balance of \$1,428,493. However, if they suffer a stock market crash and experience a return of *negative* 40%, their balance will fall to:

$$\$1,322,680 * (1 - .40) = \$793,608$$

In this case, they may only withdraw \$33,333 at the beginning of each year for the next 40 years if their retirement account earns 3% interest:

| <u>Time Value of Money</u> |              |
|----------------------------|--------------|
| <b>BGN</b>                 | BGN          |
| <b>P/Y</b>                 | 1            |
| <b>PV</b>                  | -\$793,608   |
| <b>FV</b>                  | \$0          |
| <b>N</b>                   | 40           |
| <b>I/Y</b>                 | 3%           |
| <hr/>                      |              |
| <b>PMT=</b>                | -\$33,333.37 |