

### Practice Quiz 8: Personal Taxes (Solutions)

1. Ernie earned \$75,000 during 2012. He is subject to the following marginal tax rates:

Income	Marginal Tax Rate
from \$0 to \$8,700	10%
over \$8,700 to \$35,350	15%
over \$35,350 to \$85,650	25%
over \$85,650 to \$178,650	28%
over \$178,650 to \$388,350	33%
over \$388,350	35%

Ernie qualifies for a \$3,800 personal exemption and may elect to take a \$5,950 standard deduction. In 2012, he paid \$3,000 in student loan interest and made a \$1,000 charitable contribution to his alma mater, both of which are tax deductible expenses (assume neither are above-the-line deductions). How much must Ernie pay in taxes in 2012? What is his effective tax rate?

**Ans.** Because the standard deduction is greater than his \$4,000 in itemizable deductions, Ernie may reduce his taxable income by the \$5,950 standard deduction. He may also take the \$3,800 personal exemption. Ernie's taxable income is therefore:

$$\$75,000 - \$5,950 - \$3,800 = \$65,250$$

His taxes owed are calculated as:

$$\begin{aligned} \text{Taxes} &= 0.10 * \$8,700 + 0.15 * (\$35,350 - \$8,700) + 0.25 * (\$65,250 - \$35,350) \\ &= \$12,342.50 \end{aligned}$$

His effective tax rate is then:

$$\text{Effective Tax Rate} = \frac{\$12,342.50}{\$75,000} = 16.46\%$$

2. At the beginning of the year, Mr. and Mrs. Jacobson have a current mortgage balance of \$120,000, on which they pay a 6% APR. They itemize their deductions, and would do so whether or not they had mortgage. They are subject to a tax rate of 25%. How much will the mortgage interest deduction save the Jacobsons during the year? What is the after-tax interest rate on their mortgage? (For simplicity, assume the mortgage payment is made annually.)

**Ans.** The Jacobsons will pay  $0.06 \times \$120,000 = \$7,200$  in mortgage interest in 2012. The mortgage interest deduction allows them to reduce their taxable income by this amount, allowing them to save  $0.25 \times \$7,200 = \$1,800$  in reduced taxes.

The after-tax interest rate on their mortgage is  $6\% \times (1 - 0.25) = 4.5\%$ .

3. Amy and Joe both invest \$5,000 in the stock market. Amy invests passively and holds onto her stocks for 30 years. Joe actively trades, turning over his portfolio yearly. Consequently, Amy is subjected to a deferred long-term capital gains tax of 15%, while Joe is subject to an annual short-term capital gains tax at his marginal income tax rate, which is 30%. If both earn an annual return of 8% on their investment, how much will each have in 30 years, after tax? What will be their after-tax returns? What pre-tax return must Joe achieve through active trading to match Amy's wealth in 30 years?

**Ans.** Because Amy invests for the long-term and is subject to a deferred capital gains tax of 15%, her final wealth will grow to:

$$F = P * [(1 + r)^N(1 - T) + T] = \$5,000[(1.08)^{30}(1 - 0.15) + 0.15] = \$43,516$$

And her after-tax return will be:

$$r_{after-tax} = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 = \left(\frac{\$43,516}{\$5,000}\right)^{\frac{1}{30}} - 1 = 7.48\%$$

Because Joe trades actively and turns over his portfolio every year, he is subject to a 30% accrual tax:

$$F = P[1 + r(1 - T)]^N = \$5,000[1 + .08(1 - 0.30)]^{30} = \$25,638$$

And his after-tax return will be:

$$r_{after-tax} = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 = \left(\frac{\$25,638}{\$5,000}\right)^{\frac{1}{30}} - 1 = 5.60\%$$

Amy achieves a much higher ending wealth than Joe because she is taxed at a lower effective rate. However, because Joe is trading actively, he may be able to achieve better portfolio performance than Amy through savvy stock selection. To at least match Amy's ending wealth, Joe's return must satisfy the following:

$$F = P[1 + r(1 - T)]^N = \$43,516$$

$$\rightarrow r = \frac{1}{1 - T} \left[ \left( \frac{\$43,516}{P} \right)^{\frac{1}{N}} - 1 \right] = \frac{1}{1 - 0.30} \left[ \left( \frac{\$43,516}{\$5,000} \right)^{\frac{1}{30}} - 1 \right] = 10.68\%$$

In other words, Joe must beat the market by 2.68% each year for 30 years to justify his active trading. If markets are efficient, this will be difficult to do!

4. Jason wishes to invest \$3,000 (in *after-tax* dollars) for retirement. He will do so in a stock index fund and expects an average annual return of 7%. Compare the after-tax value of Jason's contribution in 25 years if he (a) invests outside of any tax-advantaged account, (b) in a Roth IRA, and (c) in a traditional IRA. Assume a long-term capital gains tax of 15% and an income tax rate of 25% both today and when Jason withdraws in 25 years. Discuss how the after-tax value of an investment in a Roth IRA versus traditional IRA would change if Jason's tax rate in retirement is lower than it is today.

**Ans.** If Jason invests in a non-tax-advantaged account, he will be taxed at the long-term capital gains rate in 25 years:

$$F = P[(1 + r)^N(1 - T) + T] = \$3,000 * [1.07^{30}(1 - 0.15) + 0.15] = \$19,861$$

If Jason invests in a Roth IRA, his withdrawal will be tax exempt:

$$F = P(1 + r)^N = \$3,000 * 1.07^{30} = \$22,837$$

If Jason instead invests in a traditional IRA, he will make contributions with *pre-tax* income. The pre-tax equivalent to \$3,000 in *after-tax* dollars is \$4,000:

$$P_{after-tax} = P_{pre-tax}(1 - T) \rightarrow P_{pre-tax} = \frac{P_{after-tax}}{1 - T} = \frac{\$3,000}{1 - .25} = \$4,000$$

Taxes on such a contribution will then be deferred:

$$F = P(1 + r)^N(1 - T) = \$4,000(1.07)^{30}(1 - 0.25) = \$22,837$$

When the tax rate is the same today and at retirement, the future values of a Roth IRA contribution and a traditional IRA contribution are equal. If Jason's income tax rate at retirement, however, will be lower than it is today, the after-tax value of an investment in a traditional IRA is greater than that of an investment in a Roth IRA.