



THE OHIO STATE UNIVERSITY

**An Analysis of Default Risk in the
Home Equity Conversion Mortgage (HECM) Program**

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Aging in Place: Analyzing the Use of the Reverse Mortgage to Preserve Independent Living & Enable Housing Stability

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U of W: J. Michael Collins

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Research Program (2012-2016)

1. Empirical Modeling

- HECM terminations & default
- Take-up of HECMs
- HECM loan terms and withdrawal behaviors

2. Survey of Counseled Seniors

- Longer term well-being of HECM borrowers
- May 2014, targeting 5,000 respondents: (1) current HECM borrowers, (2) terminated HECM borrowers, and (3) seniors who sought counseling but did not get a reverse mortgage.

3. Post Origination Monitoring Pilot

- RCT design; financial planning and reminders after closing
- Target date: June 2014

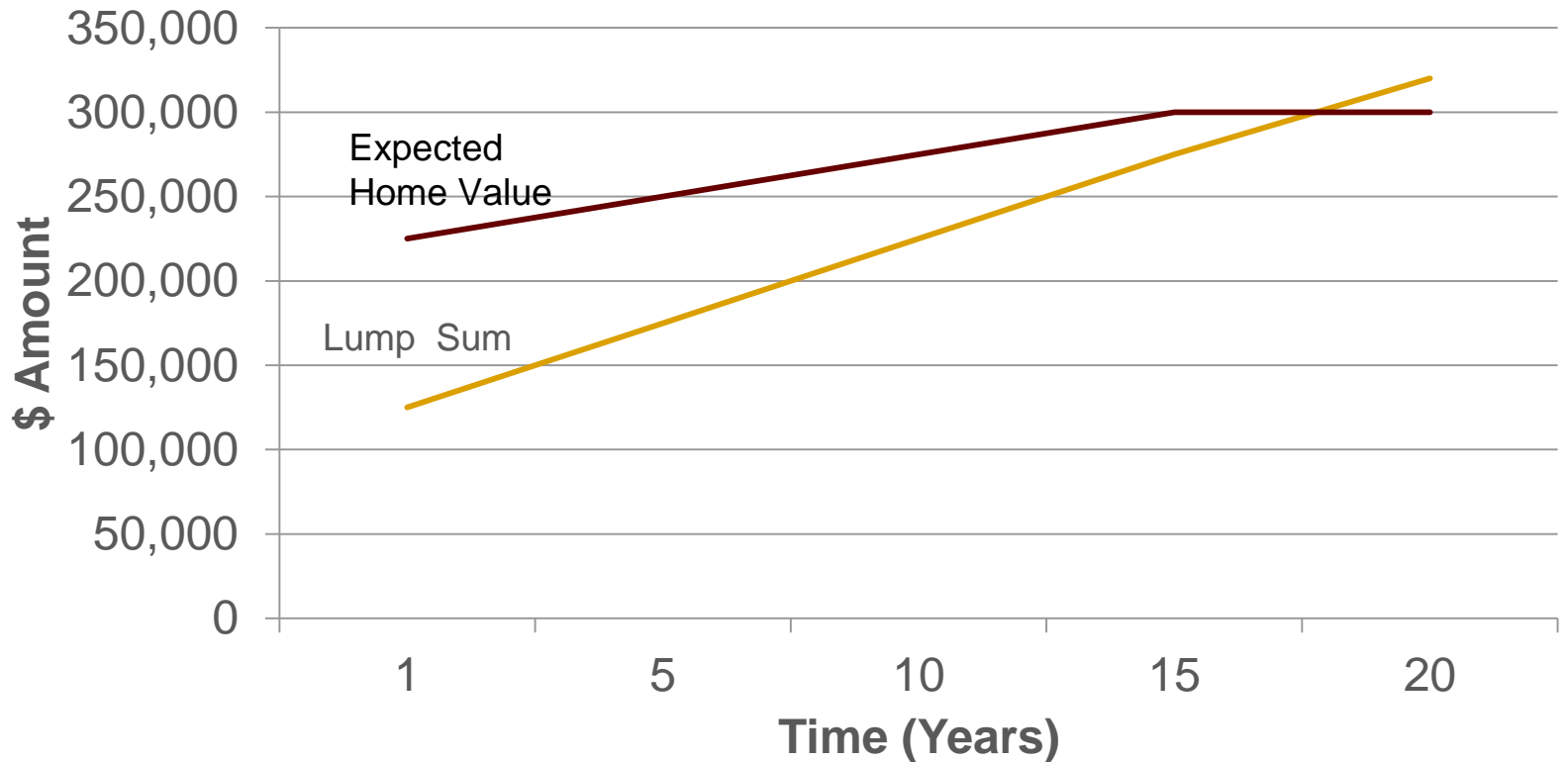


Reverse Mortgage 101

- Extract equity from the home through a mortgage that does not become due until the last borrower sells the home, moves out permanently, or dies, as long as the borrower meets the obligations of the mortgage note
 - Obligations include living in the home as primary residence, **pays property taxes, homeowners insurance**, homeowners association dues and assessments, and maintains the home.
- No payments on the loan are required during the life of the loan. Money borrowed, plus associated interest and fees, are added to the balance due that continues to grow over time (mortgage “in reverse”)
 - Line of Credit
 - Tenure or Term (similar to annuity)
 - Lump Sum Distribution
 - Some combination of the above



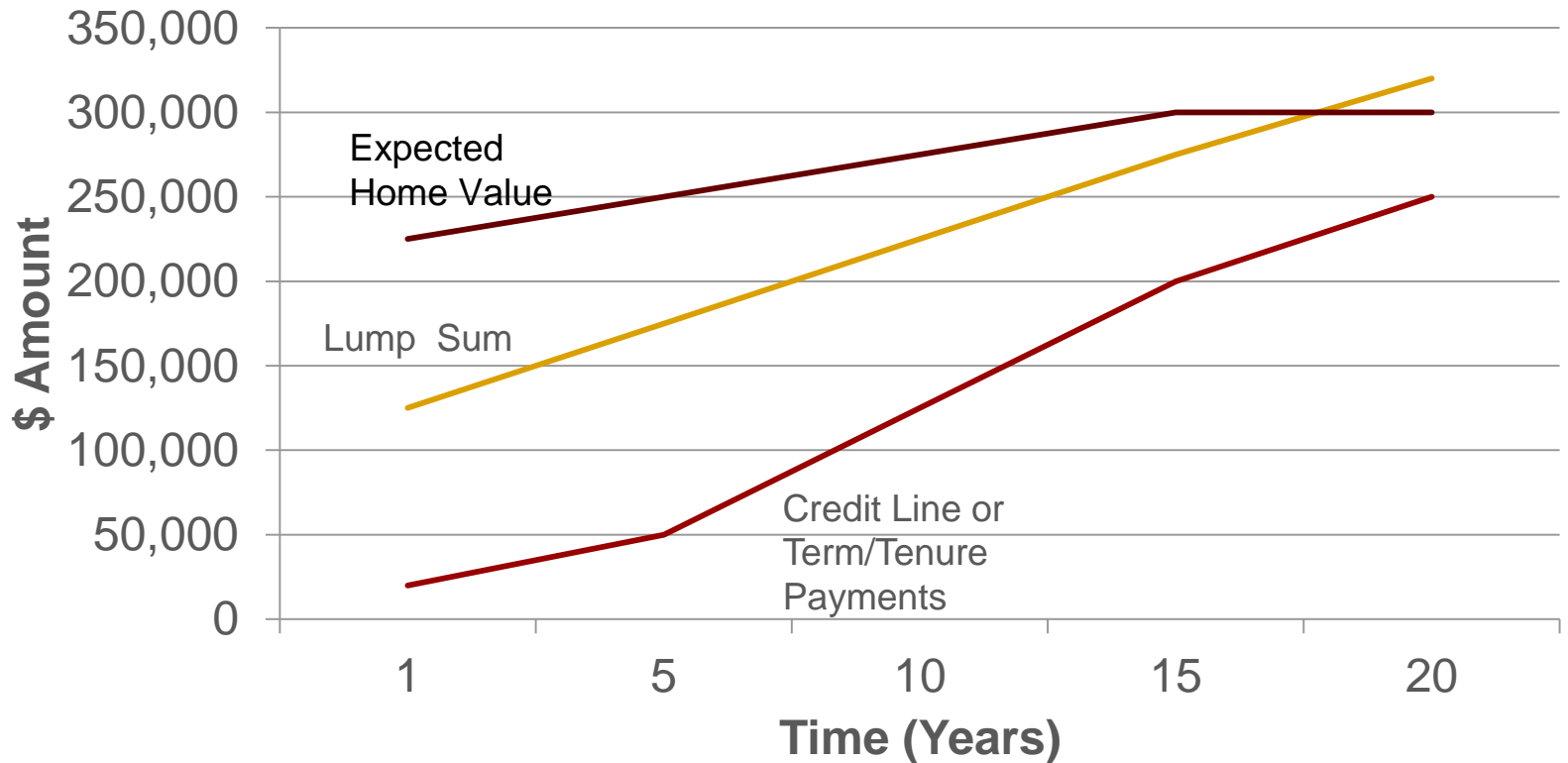
Reverse Mortgage Debt



Maximum Claim Amount (home value at closing)= \$225,000
Initial Principal Limit = \$125,000



Reverse Mortgage Debt



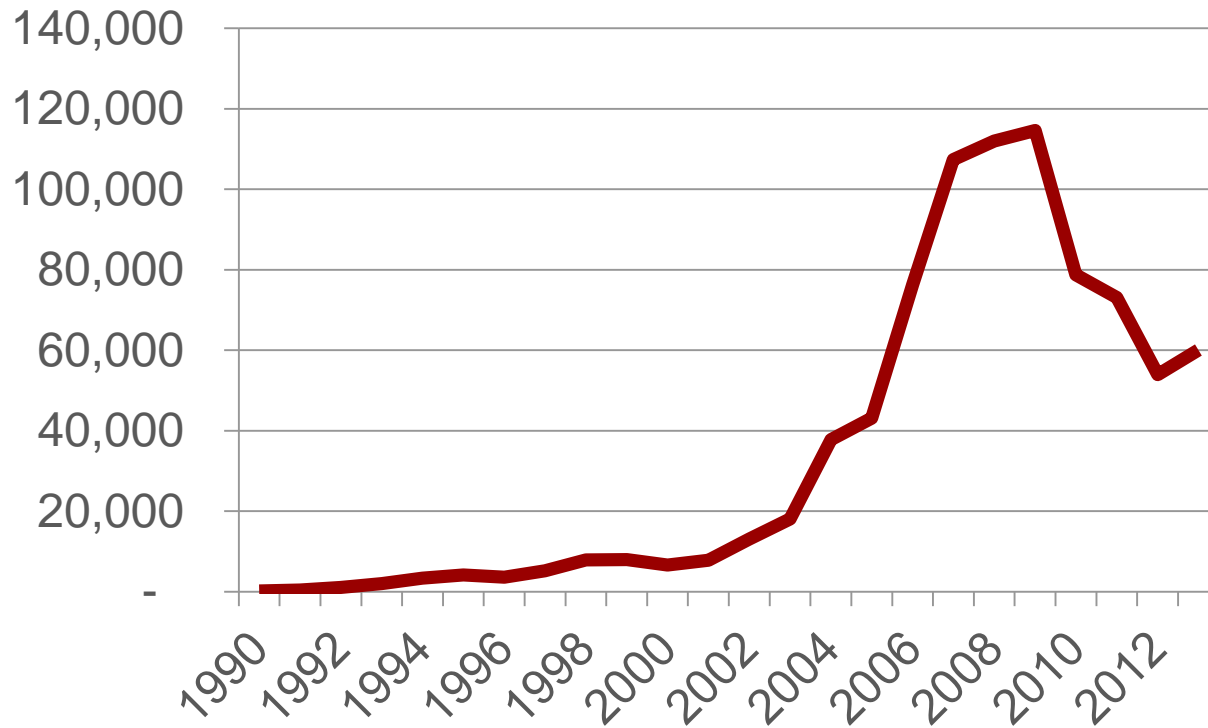
Maximum Claim Amount (home value at closing)= \$225,000

Initial Principal Limit = \$125,000



Take-Up of HECMs

Number of Loans by Year



Source: Author's calculations from HUD HECM data

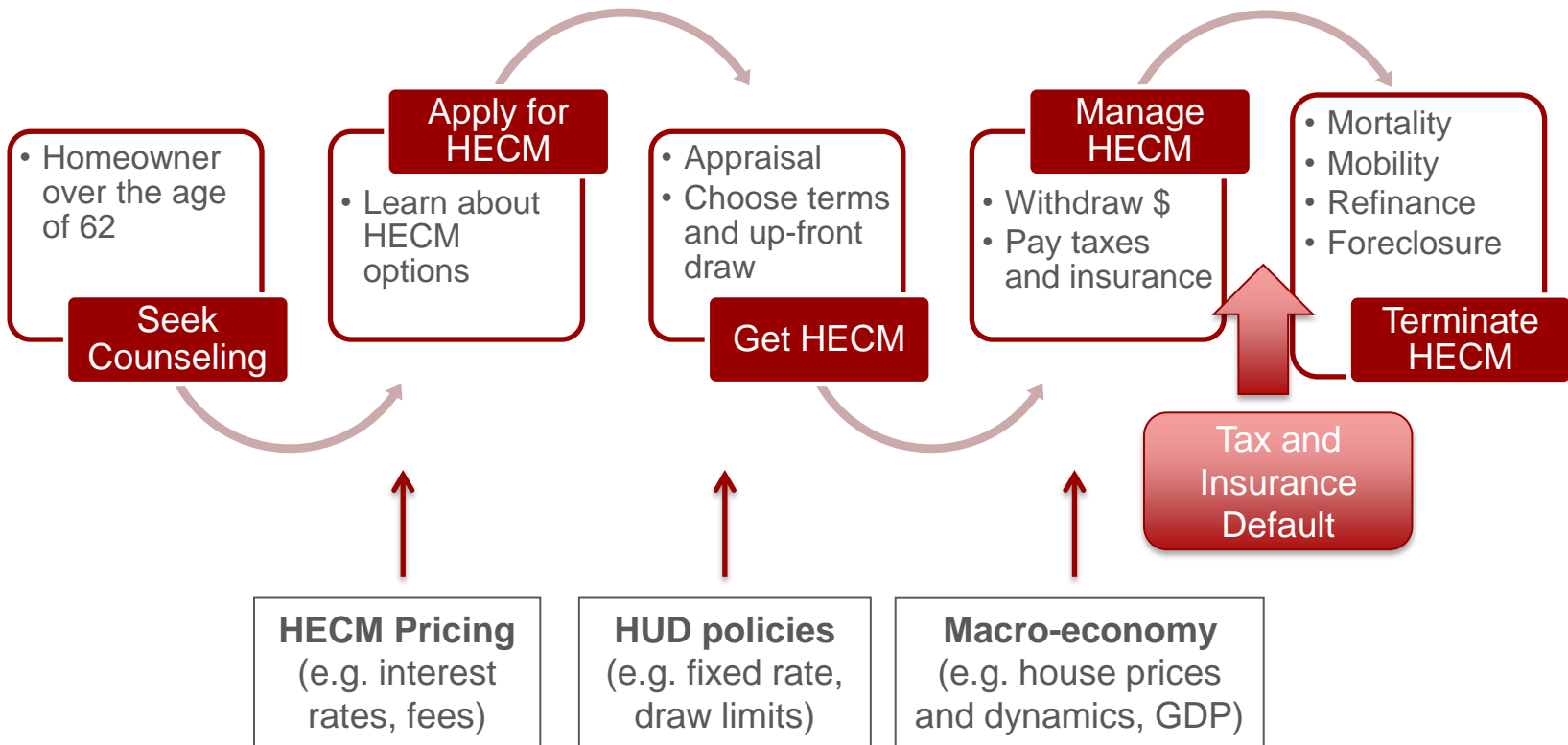


HECM Tax & Insurance Default

- **Motivation**
 - 9.4 percent of all HECM borrowers in technical default due to non-payment of property taxes and/or homeowner's insurance, as of February 2012
 - HUD policy response:
 - Limits on up-front draw %
 - Financial assessment requirement (underwriting criteria)
 - Life expectancy set-aside (LESA)
- **Explanatory factors at origination expected to be associated with default**
 - Lack of financial resources or excessive expenditures
 - Income, assets, available credit, debt burdens
 - History of poor credit performance
 - Credit score, missed installment/revolving payments, tax liens
 - Management of HECM funds
 - Initial withdraw %



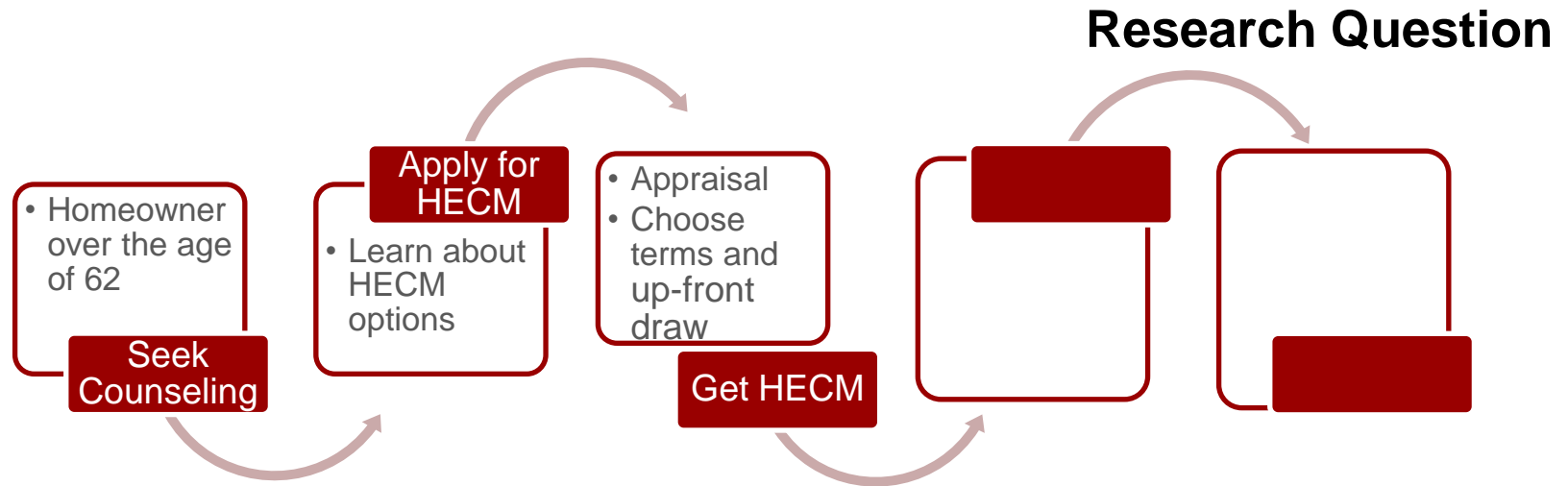
HECM Lifecycle





Previous research

- **Potential demand** for reverse mortgages
 - **General demand:** Venti and Wise 1991; Merrill, Finkel, and Kutty 1994; Rasmussen, Megbolugbe, and Morgan 1995; Mayer and Simons 1994; Costa-Font, Gil, and Mascarilla 2010
 - **Life-cycle model:** Nakajima and Telyukova 2013
- **Take-Up** of reverse mortgages
 - **General take-up:** Shan 2011
 - **House price dynamics:** Haurin et al. 2013
 - **Selection and moral hazard:** Davidoff and Welke 2004; Davidoff 2013; 2014
- **Performance** of reverse mortgages
 - **Termination outcomes:** Szymanoski, DiVenti, and Chow, 2000; Szymanoski, Enriquez, and DiVenti 2007; Rodda, Lam and Youn 2004; Bishop and Shan 2008
 - **Ruthless terminations:** Davidoff and Wetzel 2013; Davidoff 2013
 - **Pricing risks:** Szymanoski 1994; Chinloy and Megbolugbe 1994
 - **Tax and insurance default:** IFE 2011; 2012; 2013

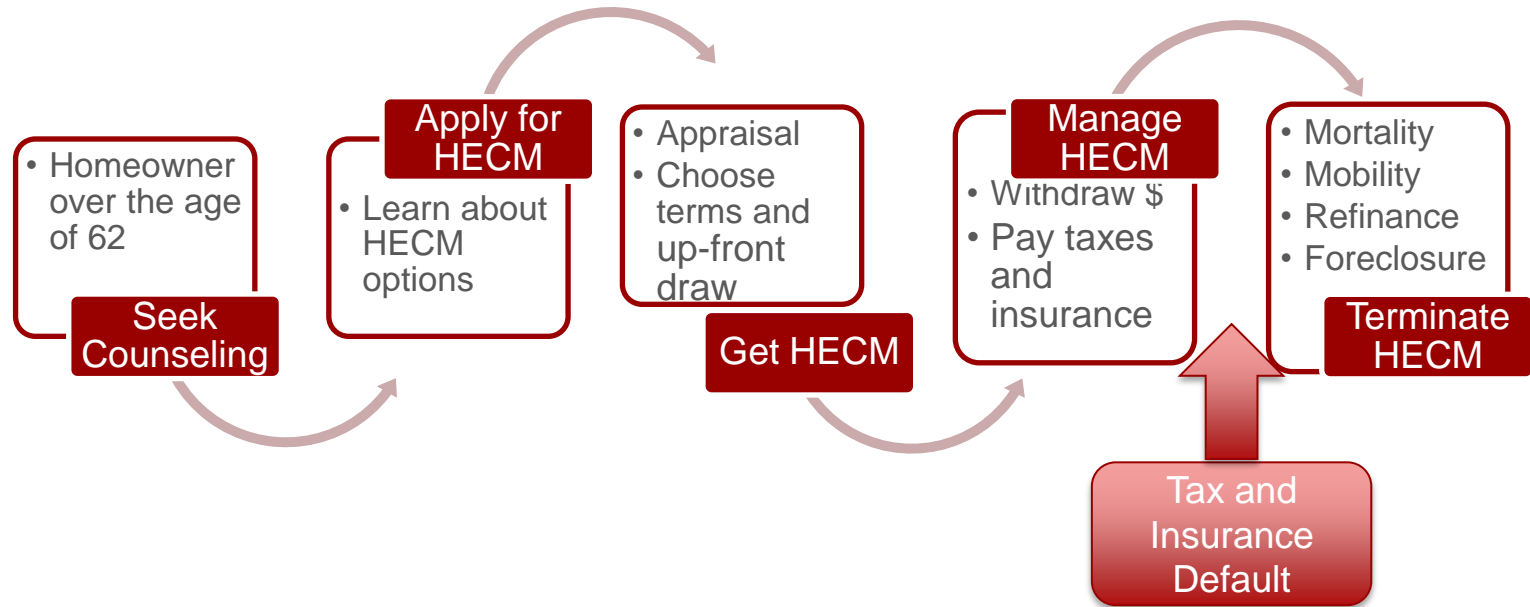


Ongoing Research Agenda:

- Who seeks counseling for a HECM?
- Of those who apply for HECMs, who gets a HECM?



Research Question



Ongoing Research Agenda:

- What factors are associated with the selection of different HECM loan terms, and how does this affect subsequent default and termination?



1. CredAbility counseling data
 - 2006 - 2011, including more than 30,000 seniors
 - NCOA's Financial Interview Tool (FIT) data after October 2010
2. Equifax credit report data
 - time of counseling & annually thereafter
3. Economic indicators
 - national, state and county level, time varying
4. HUD HECM loan data
 - includes T&I defaults

COUNSELED	HECM	T&I Default
(N=28,129)	(N=16,283)	(N=1,173)
	57.9%	7.2%



Sample Data: Demographics

Demographic Characteristics, Reverse Mortgage Counseling Clients 2006-2011

	COUNSELED (N=28,129)	HECM (N=16,283)	T&I DEFAULT (N=1,173)
	mean	mean	mean
Hispanic	11.0%	9.4%	18.8%
Race - white	63.3%	68.1%	42.4%
Race - black	16.6%	12.7%	26.3%
Race - Asian	0.9%	0.9%	0.7%
First language - not English	7.2%	5.6%	13.0%
Unmarried Male	16.1%	15.7%	20.5%
Unmarried Female	36.2%	39.3%	42.1%
Age - youngest household member	72	72	71
Education - bachelors degree	11.0%	11.1%	7.5%
Education - high school diploma	32.9%	31.0%	28.3%
Education - advanced degree	4.8%	4.6%	3.0%
Education - some college	19.8%	19.7%	13.1%

Source: CredAbility Counseling Data, 2006-2011



Sample Data: Financials

Financial Characteristics, Reverse Mortgage Counseling Clients 2006-2011

	COUNSELED (N=28,129)		HECM (N=16,283)		T&I Default (N=1,173)	
	mean	median	mean	median	mean	median
Monthly income - sum, non-missing	2,311	1,880	2,337	1,918	1,849	1,534
Taxes - property taxes/income, non-missing	0.091	0.063	0.096	0.067	0.112	0.082
Revolving account high credit - balance	20,672	5,092	23,231	7,979	7,077	83
Revolving balance/income	0.231	0.031	0.252	0.038	0.161	0.000
Installment balance/income	0.236	0	0.221	0	0.291	0

Source: CredAbility Counseling Data, 2006-2011



Sample Data: Borrower Risk

Borrower Risk Characteristics, Reverse Mortgage Counseling Clients 2007-2011

	FULL DATA (N=28,129)		HECM (N=16,283)		T&I Default (N=1,173)	
	mean	median	mean	median	mean	median
FICO score, non-missing (N=26,253)	678	690	693	709	597	593
Mortgage- foreclosure started	0.019	0	0.010	0	0.028	0
Bankruptcy - any in last 12 months	0.010	0	0.006	0.000	0.011	0
Tax lien - percent with a tax lien or judgment	0.102		0.079		0.169	
Mortgage past due, 2+ months	0.057	0	0.035	0	0.101	0

Source: CredAbility Counseling Data, 2007-2011



Sample Data: Property & Mortgage Characteristics

Property & Mortgage Characteristics, Reverse Mortgage Counseling Clients 2006-2011

	COUNSELED (N=28,129)		HECM (N=16,283)		T&I Default (N=1,173)	
	mean	median	mean	median	mean	median
Monthly mortgage payments	498	87	462	0	487	0
HELOC indicator	0.130		0.140		0.093	
Excess home value amount	18,006	0	17,220	0	12,696	0
HECM-Estimated Net IPL (Take-Up Model)	84,555	61,087	93,186	70,763	71,997	55,395
HECM-Actual Net IPL (Default Model)			83,147	62,603	63,851	46,823
HECM- Actual IPL (Withdrawal Model)			139,977	115,688	129,410	109,662
HECM- Home debt/IPL (Withdrawal Model)			0.387	0.352	0.470	0.521
Up-front draw % (Default Model)			0.771	0.939	0.883	0.934
Exposure- # days since origination as of July 1, 2012			799	675	1,118	1,140
Fixed rate policy indicator	0.781		0.736		0.477	

Source: CredAbility & HUD data, 2006-2011



Model: Truncated Bivariate Probit, with Endogenous Regressor

A household's selection into HECM is modeled as

$$HECM_i = \begin{cases} 1 & \text{if } X_i\beta_1 + S_i\gamma + u_{1i} > 0 \\ 0 & \text{otherwise} \end{cases}$$



$D_i=1$ indicates that borrower i defaults. D_i is observed only if the person is a HECM borrower: $HECM_i=1$.

$$D_i = \begin{cases} 1 & \text{if } X_i\beta_2 + Z_i\delta + W_i\alpha + u_{2i} > 0 \text{ and } HECM_i = 1 \\ 0 & \text{if } X_i\beta_2 + Z_i\delta + W_i\alpha + u_{2i} \leq 0 \text{ and } HECM_i = 1 \end{cases}$$

A household's initial withdrawal W_i is modeled as

$$W_i = X_i\beta_3 + H_i\theta + u_{3i}$$

W_i is observed only if the household obtained HECM. We estimate the three equations simultaneously (selection, withdrawal and T&I default)



Model: Truncated Bivariate Probit, with Endogenous Regressor



S_i only in selection equation	Z_i only in default equation	H_i only in withdrawal equation
Estimated Net Initial Principal Limit	Net Initial Principal Limit	Actual Initial Principal Limit
Excess of home value above MCA	% upfront draw (W_i)	Mortgage/IPL
State house price deviation from the state's long run norm	Difference between the date of origination and July 2012	Fixed rate policy dummy (=1 after Apr 1, 2009)
		Interaction between fixed rate dummy & spread between average interest rates of FRM and ARM.

X_i in all equations, includes demographic characteristics, income, property tax burden, debt burdens, FICO, credit characteristics, delinquencies, state and year fixed effects



**Truncated Bivariate Probit, HECM Take-Up
Conditional on up-front draw%, Select Significant
Results (p<.05)**

Results: HECM Take-Up

	Marginal Effects
Race- black	-0.0661
Unmarried male	0.0528
Unmarried female	0.1175
Age youngest owner	0.0200
Age youngest owner squared	-0.0001
Non-English Speaking	-0.0517
Monthly Income	0.0122
Revolving balance/income	0.0540
Mortgage payments	-0.0163
HELOC indicator	0.0175
FICO score	0.0005
Foreclosure started	-0.0701
Bankruptcy in past 12 months	-0.0996
Mortgage past due 2+ months	-0.0461
Tax lien or judgment	-0.0352
Estimated IPL net	0.0005
Excess home value	-0.0003

Fixed effects for state and year.



MLE, Up-Front Withdrawal %, Accounting for Partial Observability of HECM; Select Significant Results (p<.05)

	<i>b</i>
Hispanic	0.0238
Race - black	0.0387
Unmarried male	0.0339
Unmarried female	0.0147
Non-English speaking	0.0556
Education- college	-0.0292
Education- post graduate	-0.0324
Monthly income	0.0061
Property taxes/income	-0.2176
Revolving balance/income	0.0416
Installment balance/income	0.0137
Available revolving credit	-0.0003
FICO score	-0.0002
Mortgage past due 2+ months	-0.0306
IPL_postHECM	-0.0001
Home Debt/Actual IPL	0.3266
Policy dummy for fixed rate	0.0551

Fixed effects for state and year. Other model variables not significant at p<.05

Results: % Withdrawal

- Minority borrowers have higher initial draws than non-minority borrowers.
- Borrowers who completed education at a four year college or graduate school have about 3% lower initial draws.
- While tax liens are associated with slightly higher initial draws, a higher property tax burden is associated with taking less money out up front.
- Higher revolving and installment debt is associated with slightly higher initial draws.
- A 100 point increase in credit score is associated with a 2% decrease in the initial draw.
- A \$10,000 increase in available credit is associated with a 3% decrease in the initial draw.
- The fixed rate policy beginning in 2009 is associated with a 5.5% increase in the initial draw.



**Truncated Bivariate Probit, T&I Default
Conditional on HECM and up-front draw%,
Select Significant Results (p<.05)**

	Marginal Effects
Hispanic	0.0257
Race - black	0.0197
Unmarried male	0.0269
Monthly income	-0.0058
Property taxes/income	0.0546
Revolving balance/income	-0.0152
Available revolving credit	-0.0005
FICO score	-0.0004
Mortgage past due 2 months+	0.0224
Tax lien or judgment	0.0205
Up-front draw %	0.1251

Fixed effects for state and year, and controls for exposure days. Other model variables not significant at p<.05

Results: T&I Default

- Minority borrowers' default rates are about 2 percentage points higher than non-minority borrowers.
- A \$1,000 increase in monthly income is associated with about a ½ (.58) percentage point decrease in default rate.
- An increase in property tax to income burden is associated with increased default.
- An additional \$10,000 in available credit is associated with a ½ (.5) percentage point decrease in the default rate.
- A 100 point increase in credit score is associated with a 4 percentage point decrease in the default rate.
- Borrowers in default on their mortgage, or with tax liens or judgments have default rates that are about 2 percentage points higher.
- A 10 percentage point increase in up-front draw % is associated with a 1.2 percentage point increase in default rate.



Policy Simulations

- **Impose new up-front draw limits**
 - No mortgage debt: 60% IPL
 - If mortgage debt: payoff, up-front costs + 10% IPL
 - Simulation assumptions:
 - All still get HECMs, take lesser of observed draw or max draw limit
- **Impose credit risk thresholds & LESA affordability**
 - Apply thresholds based on credit score and credit report attributes
 - If hhld fails threshold, see if hhld could afford LESA from net IPL
 - Fail, afford LESA: get HECM, T&I default = 0
 - Fail, not afford LESA: do not get HECM (T&I default not observed)
 - Simulation assumptions:
 - LESA estimates based on 2008-2010 property tax rates
 - Threshold is hard cut-off requiring LESA
 - Those who are required to take LESA have IPL reduced by LESA \$
 - T&I default rate for those taking LESA is 0%



Policy Simulations: Initial Withdrawal Limits

Predicted Default Probability Conditional on HECM

	%Δ in Total HECM volume	Full Sample Predicted T&I Default Rate Before Policy	Full Sample Predicted T&I Default Rate After Initial Draw Limit ¹	Δ in T&I Default Rate ²	% Δ in T&I Default Rate ³
<i>HECM Sample</i>		7.20%	5.55%	-1.65%	-22.88%
<i>LESA Based on Credit Score Thresholds</i>					
Observations with credit scores LESA for Credit score less than 500 LESA for Credit score less than 580		7.03%	5.43%	-1.60%	-22.77%
<i>LESA Based on Credit Thresholds</i>					
Observations with credit reports LESA for Delinquent Mortgage/In Foreclosure LESA for Tax Lien LESA for Delinquent Installment LESA for Delinquent Revolving LESA for Any Above					



Policy Simulations: Credit Score Thresholds

Predicted Default Probability Conditional on HECM

	%Δ in Total HECM volume	Full Sample Predicted T&I Default Rate Before Policy	Full Sample Predicted T&I Default Rate After Initial Draw Limit ¹	Δ in T&I Default Rate ²	% Δ in T&I Default Rate ³
<i>HECM Sample</i>		7.20%	5.55%	-1.65%	22.88%
<i>LESA Based on Credit Score Thresholds</i>					
Observations with credit scores		7.03%	5.43%	-1.60%	-22.77%
LESA for Credit score less than 500	-1.07%		4.55%	-2.48%	-35.30%
LESA for Credit score less than 580	-4.45%		2.90%	-4.13%	-58.75%
<i>LESA Based on Credit Thresholds</i>					
Observations with credit reports					
LESA for Delinquent Mortgage/In Foreclosure					
LESA for Tax Lien					
LESA for Delinquent Installment					
LESA for Delinquent Revolving					
LESA for Any Above					



Policy Simulations: Credit Score Thresholds

Predicted Default Probability Conditional on HECM

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<i>LESA Based on Credit Score Thresholds</i>					
Observations with credit scores		7.03%	5.43%	-1.60%	-22.77%
LESA for Credit score less than 500	-1.07%		4.55%	-2.48%	-35.30%
LESA for Credit score less than 580	-4.45%		2.90%	-4.13%	-58.75%
<i>LESA Based on Credit Thresholds</i>					
Observations with credit reports		6.76%	5.18%	-1.58%	-23.43%
LESA for Delinquent Mortgage/In Foreclosure	-2.09%		4.26%	-2.5%	-37.03%
LESA for Tax Lien	-2.26%		4.22%	-2.54%	-37.64%
LESA for Delinquent Installment	-0.66%		4.89%	-1.87%	-27.63%
LESA for Delinquent Revolving	-2.71%		3.94%	-2.82%	-41.71%
LESA for Any Above	-5.64%		2.74%	-4.02%	-59.46%



Policy Implications & Conclusions

- Policy viability of HECM program
 - T&I defaults that result in foreclosure can contribute to fiscal insolvency of the MMI fund
 - “Headline risk” of program and perceived public value
- Mitigating default risk while not (overly) restricting access
 - Restrictions on initial withdrawals
 - Credit risk thresholds & LESA affordability
- Next steps:
 - Generalizing empirical model
 - Other outcomes of consumer well-being
 - Post-origination monitoring as innovation to reduce default



Questions?



Hypotheses

Variable	Withdrawal %	Default
Financial Resources & Expenditures		
Income	-	-
Borrowing capacity	-	-
Property tax burden	?	+
Debt ratios	+	+
Borrower Credit Risk		
Credit score	-	-
Tax liens	+	+
Missed mortgage payments	?	+
Management of HECM Funds		
Initial withdrawal %		+
Net IPL		-
Home debt/IPL	+	
Fixed rate, full draw policy	+	



Truncated Bivariate Probit with Endogenous Regressor

- HECM Selection

$$y_{i1}^* = x'_{i1}\beta_1 + z'_i\alpha_1 + \epsilon_{i1} \quad (1)$$

The household selects HECM ($y_{i1} = 1$) if $y_{i1}^* > 0$. Among HECM borrowers, initial withdrawal as a percentage of initial principal limit (w_i) and whether the borrower has defaulted on tax or insurance (y_{i2}) are observed.

- T&I default

$$y_{i2}^* = x'_{i2}\beta_2 + z'_i\alpha_2 + w_i\gamma_2 + \epsilon_{i2} \quad (2)$$

The household defaults ($y_{i2} = 1$) if $y_{i2}^* > 0$ and $y_{i1} = 1$.

- Withdrawal¹

$$w_i = x'_{i3}\beta_3 + z'_i\alpha_3 + \epsilon_{i3} \quad (3)$$



Truncated Bivariate Probit with Endogenous Regressor

In Eqs. (1)-(3), z_i are common variables, x_{i1} , x_{i2} , x_{i3} are unique to each equation respectively. The unobservables $[\epsilon_{i1} \ \epsilon_{i2} \ \epsilon_{i3}]$ are jointly normal with mean 0 and variance

$$\Sigma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13}\sigma \\ \rho_{12} & 1 & \rho_{23}\sigma \\ \rho_{13}\sigma & \rho_{23}\sigma & \sigma^2 \end{bmatrix} \quad (4)$$

The unobservables are assumed to be independent from x_{i1}, x_{i2}, x_{i3} and z_i . The withdrawal w_i is correlating with ϵ_{i2} if ϵ_{i3} is correlated with ϵ_{i2} .

There are 3 cases.

	case 1	case 2	case 3	
y_{i1}	1	1	0	HECM take-up
y_{i2}	1	0	.	t&i default
w_i	observed	observed	.	initial withdrawal



Truncated Bivariate Probit with Endogenous Regressor

- Case 1: the household selects HECM $y_{i1} = 1$, withdraws w_i , and defaults $y_{i2} = 1$. The joint density is

$$\begin{aligned}
 l_{i1}(\theta) &= f(y_{i1} = 1, y_{i2} = 1, w_i = w | x_{i1}, x_{i2}, x_{i3}, z_i) \\
 &= \int_{-x'_{i1}\beta_1 - z'_i\alpha_1} \int_{-x'_{i2}\beta_2 - z'_i\alpha_2 - w_i\gamma} \phi_3(\epsilon_1, \epsilon_2, w - x'_{i3}\beta_3 - z'_i\alpha_3) d\epsilon_1 d\epsilon_2
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 &= \int_{-x'_{i1}\beta_1 - z'_i\alpha_1} \int_{-x'_{i2}\beta_2 - z'_i\alpha_2 - w_i\gamma} \phi_{\epsilon_1, \epsilon_2 | \epsilon_3 = w - x'_{i3}\beta_3 - z'_i\alpha_3}(\epsilon_1, \epsilon_2) f(w | x_{i3}, z_i) d\epsilon_1 d\epsilon_2
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 &= f(w | x_{i3}, z_i) \int_{-x'_{i1}\beta_1 - z'_i\alpha_1} \int_{-x'_{i2}\beta_2 - z'_i\alpha_2 - w_i\gamma} \phi_{\epsilon_1, \epsilon_2 | \epsilon_3 = w - x'_{i3}\beta_3 - z'_i\alpha_3}(\epsilon_1, \epsilon_2) d\epsilon_1 d\epsilon_2 \\
 &= f(w | x_{i3}, z_i) P(y_{i1} = 1, y_{i2} = 1 | x_{i1}, x_{i2}, x_{i3}, z_i, w_i = w)
 \end{aligned} \tag{7}$$



Truncated Bivariate Probit with Endogenous Regressor

In Eq. (5), ϕ_3 is the density of trivariate normal distribution with mean $[0 \ 0 \ 0]$ and variance Σ as in Eq. (4). Then in Eq. (6), the trivariate normal density is written as a product of the marginal density of ϵ_{i3} and the conditional density of $\epsilon_{i1}, \epsilon_{i2}$ on $\epsilon_{i3} = w - x'_{i3}\beta_3 - z'_i\alpha_3$. The terms in Eq. (7) are

$$\log f(w_i = w | x_{i3}, z_i) \propto -\frac{1}{2} \log \sigma^2 - \frac{1}{2\sigma^2} (w - x'_{i3}\beta_3 - z'_i\alpha_3)^2,$$

$$\log P(y_{i1} = 1, y_{i2} = 1 | x_{i1}, x_{i2}, x_{i3}, z_i, w_i = w) = \log \Phi_2(x'_{i1}\beta_1 + z'_i\alpha_1, x'_{i2}\beta_2 + z'_i\alpha_2 + w_i\gamma_2; \bar{\mu}_{i,1}, \bar{\Sigma}_1)$$

where $\Phi_2(\cdot, \cdot; \bar{\mu}_{i,1}, \bar{\Sigma}_1)$ is the cdf of a bivariate normal $(\bar{\mu}_{i,1}, \bar{\Sigma}_1)$. Using the property of multivariate normal distribution,

$$\bar{\mu}_{i,1} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \Sigma_{12} \Sigma_{22}^{-1} (w_i - x'_{i3}\beta_3 - z'_i\alpha_3) = \begin{pmatrix} -\frac{\rho_{13}}{\sigma} \\ -\frac{\rho_{23}}{\sigma} \end{pmatrix} (w_i - x'_{i3}\beta_3 - z'_i\alpha_3),$$

$$\bar{\Sigma}_1 = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} = \begin{pmatrix} 1 - \rho_{13}^2 & \rho_{12} - \rho_{13}\rho_{23} \\ \rho_{12} - \rho_{13}\rho_{23} & 1 - \rho_{23}^2 \end{pmatrix},$$

$$\text{where } \Sigma_{11} = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{pmatrix}, \Sigma_{22} = \sigma^2, \Sigma_{12} = \begin{pmatrix} \rho_{13}\sigma \\ \rho_{23}\sigma \end{pmatrix}.$$



Truncated Bivariate Probit with Endogenous Regressor

The full likelihood function is

$$\log L_n(\theta) = \sum_{i=1}^n \{I(y_{i1} = 1, y_{i2} = 1) \log l_{i1}(\theta) \\ + I(y_{i1} = 1, y_{i2} = 0) \log l_{i2}(\theta) + I(y_{i1} = 0) \log l_{i3}(\theta)\}$$

In the maximum likelihood estimation, ρ_{12} , ρ_{13} , ρ_{23} and σ are not directly estimated. Directly estimated is a transformation of these parameters, $\log \sigma$ for σ and $\text{atanh} \rho = \frac{1}{2} \log \left(\frac{1+\rho}{1-\rho} \right)$ for ρ . We have $\rho = \frac{-1 + \exp(2\text{atanh} \rho)}{1 + \exp(2\text{atanh} \rho)}$. The parameter space of the transformed variable is unrestricted. The same transformation is used by the Stata routine “heckman”.